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CONVECTIVE FLOWS OF VISCOUS FLUID IN SPHERICAL LAYERS. CERTAIN ASTROPHYSICAL APPLICATIONS.

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16. Abstract

This report surveys the studies investigating the convective stability of a viscous liquid in spherical layers taking into consideration rotation, the latitudinal temperature gradient, and shear flow. The results of calculating nonlinear convective motion in spherical layers are examined.

A discussion is given of the applicability of the results obtained to studying convection in astrophysical objects.

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INTRODUCTION

Thermal convection is a phenomenon which is very common in astro- and geophysical conditions. Energy transport by convection occurs in the nuclei of stars of the upper part of the main sequence and in the shells of the stars of its lower section, particularly on the Sun. The interaction of convection with radical pulsations and rotations is apparently the mechanism which causes the cepheid variable. Let us assume that the band structure of the surface of the planets, giants Jupiter and Saturn, is the reflection of the convective instability of the atmospheres of these planets. On the Earth, convection influences the global motion in the atmosphere and the oceans, and convection in the mantle of the Earth is responsible for the motion of the continents. /*3

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Many studies have investigated convection under astro- and geophysical conditions. However, the problem is so complex that we can only think about solving certain particular problems.

The classical problem of Raleigh-Benar has been known for a long time. It covers the convective stability of an infinite horizontal layer of an incompressible liquid, in which a vertical temperature gradient is maintained [1]. However, the convection occurring under the conditions in which we are interested is a /4 much more complex phenomenon. It is basically related to the fact that convection is not an individual, isolated process, but occurs in interaction with other phenomena of stellar or planetary dynamics and thermodynamics. Thus, in many astrophysical cases, it is necessary to consider the influence of rotation and magnetic fields upon convection. Layers which are convectively unstable in real objects usually are connected with thermally stable layers. The penetration of convective pulsations in these layers must be considered in order to obtain the correct physical picture.

Frequently, a vertical temperature gradient which causes convection arises in moving media; in this case it is important to consider the interaction of convection with the shear velocity fields.

Convection is not a unique process which performs the transport of heat in stars or planets. Frequently, along with convection, radiation involves the transport of a section of the heat flow, by a certain method of changing convection.

Large density gradients in astrophysical convective zones (thus, on the Sun the density changes by 6 orders of magnitude in the convective shell, and on Jupiter -- by approximately 3-4 orders of magnitude) make it impossible to use a very simplified study of convection with the Boussinesq approximation for incompressible liquids (see, for example, [1]).

In order to study the global convective phenomena under astrophysical conditions, it is necessary to consider the spherical geometry of the layers in which it occurs. Finally, we must note one of the very important factors which complicate the calculation of the convective zones in astro- and geophysics: convection is always turbulent in them due to the enormous linear dimensions of these zones.

Examples of special effects which complicate the convective motion in astrophysical objects may be given later on, but it is sufficient to illustrate the difficulties arising in a study of convection under such conditions. A more detailed discussion of the specific nature of astrophysical convection in stars and the Sun may be found in [2,3]. /5

In this report, we discuss the influence of only individual factors upon convection, but in a more rigorous mathematical formulation. This influences the convection of spherical geometry. Secondly, it influences rotation and partially shear flow and the nonuniform distribution of temperature at the layer boundaries. A discussion of the influence of certain other effects may be found in the report by Spiegel [2] and the studies [3-8].

Section 1. Development of convective instability in spherical layer at rest.

Let us begin by examining the problem of convective stability of a viscous, incompressible liquid in a spherical layer of the thickness h, located between two concentric, spherical surfaces with the radii $r_i = r_o h$ and $r_e = (r_o + 1)h$. A spherically symmetric radial gravitation field $-f(r)\bar{r}$ acts upon the liquid (below, we shall use the spherical system r, ϑ, φ). The temperature constants at the boundaries of the layers T_i and T_e and the heat sources of constant intensity q, which are distributed uniformly in the liquid, create a radial temperature gradient ∇T. When the gradient is small, the liquid is at rest, and the temperature distribution is determined by the heat conductivity

$$\nabla T = -2\bar{r}\left(\beta_2 + \frac{\beta_1}{2r^3}\right) = -\frac{\bar{r}}{r} b_0 b(r), \tag{1.1}$$

where $\beta_2 = q/6\varkappa,\ \beta_1 = \left[T_i - T_e - \frac{q}{6\varkappa}(2r_0+1)\right] r_0(r_0+1),\ \varkappa$ -- heat conductivity coefficient assumed to be constant.

When ∇T is large enough, the heat conductivity mode becomes /6
unstable, and convection arises.

The linear problem of the development of convection in a spherical layer at rest was first examined in the studies of Chandrasekhar [4]. The dimensionless equations for small equilibrium perturbations in the Boussinesq approximation have the form:

$$\frac{\partial u}{\partial t} = -\nabla p + \Delta u + Ra\,\bar{r}\,\gamma(r)\theta,$$

$$Pr\frac{\partial \theta}{\partial t} + \bar{u}\,\nabla T = \Delta\theta, \quad \operatorname{div}\bar{u} = 0. \tag{1.2}$$

Here $\bar{u}, p$ and θ -- velocity, pressure and temperature of the perturbed motion, ∇T -- temperature gradient determined by the heat conductivity; the length, time, velocity, pressure and temperature are used as characteristic scales and are represented by

$$h,\ h^2/\nu,\ \nu/h,\ \rho_0\nu^2/h^2,\ B_0 h\,Pr$$

The following notation is introduced:

$Pr = \nu/\chi$ -- Prandtl number,

$Ra = \alpha g_0 B_0 h^4/\nu\chi$ -- the Rayleigh number;

α, ν and χ -- coefficients of volumetric expansion, viscosity and heat conductivity

ρ_0, B_0 and g_0 -- density, temperature gradient, and acceleration of gravity at the external layer boundary,

$\gamma(r) = f(r)/g_0$

It is apparent that at the layer boundaries the following conditions are satisfied:

$$u_r = \theta = 0 \quad \text{at} \quad r = r_0 \quad \text{and} \quad r = r_0 + 1 \tag{1.3}$$

If the boundaries are solid, then it is necessary that the remaining velocity components also vanish:

$$u_\vartheta = u_\varphi = 0. \tag{1.4}$$

The corresponding viscous tangential stresses must vanish at the free boundaries:

$$\tau_{r\varphi} = \tau_{r\vartheta} = 0. \tag{1.5}$$

Let us give the exponential dependence of the perturbations /7

on time: $u, p, \theta \sim \exp st$.

In order to exclude the continuity equation, it is convenient to use the general representation of a solenoidal velocity field in the form of the sum of the poloidal and toroidal components:

$$\bar{u} = \mathrm{rot}^2 s\bar{r} + \mathrm{rot}\, w\bar{r}. \tag{1.6}$$

Applying the operations $\bar{r}\,\mathrm{rot}$ and $\bar{r}\,\mathrm{rot}^2$ to the first equation (1.2), we obtain:

$$L^2(\Delta - \sigma)w = 0,$$
$$L^2(\Delta - \sigma)\Delta s - Ra\,\gamma(r)L^2\theta = 0, \qquad (1.7)$$
$$Pr\,\sigma\theta + \bar{u}\nabla T - \Delta\theta = 0.$$

Here we use L^2 to designate the operator

$$L^2 = -\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial}{\partial\mu} - \frac{1}{1-\mu^2}\frac{\partial^2}{\partial\varphi^2} \qquad (\mu = \cos\vartheta).$$

The boundary conditions in terms of the scalars of the poloidal S and toroidal W of the fields may be written in the form:

$$\theta = s = \frac{\partial s}{\partial r} = w = 0 \quad \text{on the solid boundary,}$$
$$\theta = s = \frac{\partial^2 s}{\partial r^2} = \frac{\partial w}{\partial r} - \frac{w}{r} = 0 \quad \text{on the free boundary} \qquad (1.8)$$

The spherical symmetry of the problem makes it possible to expand the solution in spherical harmonics

$$y_\ell^m(\vartheta,\varphi) = P_\ell^m(\mu)e^{im\varphi}$$

(P_ℓ^m -- associated Legendre functions). We set

$$w(r,\vartheta,\varphi) = W_\ell^m(r)\,y_\ell^m(\vartheta,\varphi), \text{ etc.}$$

Then we obtain the following equation for radial perturbation amplitudes:

$$(\mathcal{D}_\ell - \sigma) W = 0,$$
$$\mathcal{D}_\ell(\mathcal{D}_\ell - \sigma) S - Ra\,\gamma(r)\Theta = 0, \qquad (1.9) \qquad /8$$
$$(\mathcal{D}_\ell - Pr\,\sigma)\Theta + \frac{\ell(\ell+1)}{r} B(r) S = 0.$$

where the differential operator D_e is determined as

$$\mathcal{D}_\ell = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{\ell(\ell+1)}{r^2},$$

and the indices $\underline{l}$ and m are omitted here and below.

Thus, the problem of the convective stability of equilibrium is reduced to the problem of the eigenvalues for the Rayleigh number Ra.

It is known from (1.9) that W = 0, i.e., the convective flow arising at the stability limit is poloidal. We should note the following singularity of the problem being considered: The azimuthal wave number m is not included in the boundary value problem (1.8) - (1.9). For this reason, the critical Rayleigh numbers will depend only on the wave number $\underline{l}$, but not on m, i.e., the following degeneracy holds: $2\ell+1$ perturbations correspond to one and the same critical Rayleigh number with a different angular dependence $P_e^m(\mu)e^{\pm im\varphi}$ $(m \leq \ell)$. A situation arises which is similar to that which occurs in the plane layer. An infinite set of solutions corresponds to the same Rayleigh number, since Ra depends only on the square of the modulus of the horizontal wave vector $a^2 = a_x^2 + a_y^2$, , and the relationship between a_x and a_y is arbitrary [1,4].

When the following condition is satisfied

$$\frac{d}{dr}\frac{r\gamma(r)}{\beta(r)} = 0 \qquad (1.10)$$ /9

the problem (1.8) - (1.9) is self-conjugate, and consequently the principle of the stability displacement $\sigma = 0$ is valid and the convection which arises at the stability limit is stationary. Numerical calculations later performed by Durney [9] and Young [10] showed that the principle of the stability displacement is apparently valid not only under the condition (1.10), but also in the more general case of convection development in a quiet spherical layer. In the particular case $\gamma(r) = \beta(r)r^{-1} = (r_0+1)^{-1}$ the system (1.9) assumes the form:

$$\begin{aligned} &\mathcal{D}_\ell^2 S - Ra_0(r_0+1)^{-1}\Theta = 0, \\ &\mathcal{D}_\ell \Theta + \ell(\ell+1)(r_0+1)^{-1} S = 0, \\ &\mathcal{D}_\ell W = 0. \end{aligned} \qquad (1.11)$$

We shall discuss briefly the method of solving the system (1.11) with the boundary conditions (1.8), since it is characteristic for solving the more general problems of liquid motion in spherical layers. In many cases it is convenient to use the Galerkin method expanding the radial amplitudes in series with respect to the Chandrasekhar functions

$$C_\ell(\alpha_i r) = J_{-(\ell+1/2)}(\alpha_i r_0)\, J_{\ell+1/2}(\alpha_i r) - J_{\ell+1/2}(\alpha_i r_0)\, J_{-(\ell+1/2)}(\alpha_i r),$$

which represents a combination of Bessel functions of half-integral order and which satisfy the conditions $C_l(\alpha_i r_0)=0$ and $C_l(\alpha_i(r_0+1))=0$, where α_i are the roots of the equation

$$C_l(\alpha_i(r_0+1))=0. \qquad (1.12)$$

It is known [4] that equation (1.12) has an infinite set of real simple roots, and the functions $C_l(\alpha_i r)$ form a complete orthogonal system of functions with the integral property /10

$$\int_{r_0}^{r_0+1} r\, C_l(\alpha_i r)\, C_l(\alpha_j r)\, dr = N_i^l \delta_{ij}; \quad N_i^l = \frac{2}{\pi^2 \alpha_i^2}\left\{\frac{J^2_{l+1/2}(\alpha_i r_0)}{J^2_{l+1/2}(\alpha_i(r_0+1))} - 1\right\}.$$

Perturbations of temperature and the toroidal component of the velocity field, when it differs from zero, may be represented in the form of a series with respect to the Chandrasekhar functions, and the poloidal component of the velocity field in the following form:

$$S = \sum_j C_{l+1/2}(\alpha_j r)/\alpha_j^4 r^{1/2} + B_l^j r^l + \mathcal{D}_l^j r^{l+2} + \mathcal{F}_l^j r^{-(l+1)} + K_l^j r^{-(l-1)},$$

where the constants $B_l^j, \mathcal{D}_l^j, \mathcal{F}_l^j$ and K_l^j are selected so as to satisfy the four boundary conditions applied to the function S. Then the problem may be solved according to the well-known procedure.

The critical Rayleigh numbers for four types of boundary conditions are found in [4] and given in the table, together with the

TABLE

CRITICAL RAYLEIGH NUMBERS AND WAVE NUMBERS IN LAYERS OF DIFFERING THICKNESS AND FOR DIFFERENT BOUNDARY CONDITIONS

Boundary conditions \ Layer thickness		τ_0 = 0,25 δ = 4	0,43 2,33	0,67 I,5	I I	I,5 0,67	4 0,25
Free boundaries	ℓ_c	I	2	2	3	4	I0
	Ra_c	$2,I94 \cdot I0^3$	$I,708 \cdot I0^3$	$I,4I4 \cdot I0^3$	$I,20 \cdot I0^3$	$I,043 \cdot I0^3$	$8,I66 \cdot I0^2$
Lower boundary rigid upper free	ℓ_c	2	2	3	4	5	I2
	Ra_c	$2,545 \cdot I0^3$	$2,II5 \cdot I0^3$	$I,926 \cdot I0^3$	$I,739 \cdot I0^3$	$I,553 \cdot I0^3$	$I,305 \cdot I0^3$
Lower boundary free upper rigid	ℓ_c	2	2	3	4	5	I2
	Ra_c	$4,805 \cdot I0^3$	$3,6II \cdot I0^3$	$2,926 \cdot I0^3$	$2,408 \cdot I0^3$	$I,989 \cdot I0^3$	$I,456 \cdot I0^3$
Rigid boundaries	ℓ_c	2	2	3	4	6	I3
	Ra_c	$5,23 \cdot I0^3$	$4,4II \cdot I0^3$	$3,625 \cdot I0^3$	$3,II6 \cdot I0^3$	$2,7I6 \cdot I0^3$	$2,I26 \cdot I0^3$

/11

values of the critical wave numbers. It may be seen from the table that layers with more rigid boundaries are more stable, and the convective cells in them have a smaller size than in layers with free boundaries. A decrease in the layer thickness $\delta = h/r_0, h = r_0^{-1}$ leads (see the table) to a decrease in the dimensions of the convective cells, i.e., to an increase in the critical wave number l.

For a study of convection in the atmospheres of the planets and in convective shells of stars like the Sun, of greatest interest are the rules for convective flows in thin layers $\delta \leq 0.25$ with free boundaries or with a rigid lower and free upper boundary.

Within the limit of very thin layers at $\delta \to 0 \ (r_0 \gg 1)$ the system (1.11) may be reduced to a system of equations for convection in an infinite horizontal layer [1, 4]. As is known, in this case for a layer with free boundaries, the critical Rayleigh number $Ra_0 = 27\pi^4/4$, the horizontal wave number $a^2 = \frac{l^2}{r_0^2} = \frac{\pi}{2}$ and the radial amplitudes do not depend on the wave number l: /12

$$S(r) = r_0^{-1} \sin \pi (r - r_0); \ \theta(r) = 1/3 \sin \pi (r - r_0).$$

Section 2. Influence of rotation, shear, and latitudinal temperature gradient upon convection in a slowly rotating layer

For problems of astrophysics, there has been great interest in the development of convection in moving rotating media with non-uniform temperature distribution at the boundary. This formulation of the problem occurs, for example, when modeling the global hydrodynamic movements in atmospheres of large planets, particularly

Jupiter [11].

Actually, the existence of an internal heat source for Jupiter [12] must lead to convective instability of the lower part of its atmosphere and to establishment of a turbulent convection mode, since the Rayleigh numbers are enormous.

However, the data from observations of the visible surface of Jupiter (the ordered band-like structure) point to the highly organized nature of large-scale movements in the atmosphere of the planet (see, for example,[13]).

It must be assumed that large-scale convection having the form of axisymmetric rolls develops on the background of comparatively small-scale turbulent convection, which may be taken into consideration by the introduction of the effective viscosity coefficients.

If the atmosphere of the planet is not comparatively dense, then the absorption of solar radiation plays an important role, which may cause both shear zonal flows in the atmosphere and heating /13 which is not uniform with respect to latitude [11].

The study [11] examined convective stability of shear flow of a viscous liquid in a slowly rotating spherical layer with a latitudinal temperature gradient on the external boundary. For simplicity, it was assumed that the flow arises due to the difference in the angular rotational velocities Ω and $\Omega(1 + \varepsilon)$ of the internal and external boundaries of the layer and the latitudinal temperature gradient. It is assumed that the temperature is constant on the internal boundary. The latitudinal temperature gradient which simulates the heating of the planetary atmosphere by solar radiation is assumed to be symmetric with respect to the equator, i.e., the temperature on the external boundary may be represented in the form of a series in terms of even Legendre polynomials

$$T_e(\vartheta) = \sum_n T^{(2n)} P_{2n}(\mu).$$

It is apparent that the temperature T_e which is average on the external boundary equals $T^{(o)}$. Let us examine the case

$$T_e = \bar{T}^{(o)} + \bar{T}^{(2)} \bar{P}_2(\mu).$$

Due to two factors (differences of angular velocities of sphere rotation and latitudinal gradient), a stationary flow arises which may be described by the following dimensionless equations in a system of coordinates rotating at the angular velocity Ω,

$$(\bar{V}\nabla)\bar{V} = -\nabla P + \Delta\bar{V} + Pr^{-1} Ra\,\delta(r)\bar{r}\,T - 2Re(\bar{z}_0 \times \bar{V}), \qquad (2.1)$$

$$Pr\,\bar{V}\nabla T = \Delta T + q; \quad div\,\bar{V} = 0$$

with the boundary conditions

$$r = r_0 : \quad \bar{V} = T = 0;$$

$$r = r_0 + 1 : \quad \bar{V} = \varepsilon\,Re(r_0+1)\sin\vartheta\,\bar{\varphi}_0, \; T = -1 - \lambda P_2(\mu). \qquad (2.2)$$

Here:

$\bar{V}$, P and T -- velocity, pressure and flow temperature, /14

Re = $\Omega h^2/\nu$ -- Reynolds number, $\lambda = T^{(2)}/T^{(o)}$, $\bar{z}_0$ -- unit vector along the axis of rotation. When deriving the equations, it was assumed that the relationship of centrifugal force to the gravitational force is small, and ∇P includes all the potential forces.

The stationary flow (2.1) - (2.2) may be unstable with respect to small perturbations at large ε Re numbers -- shear instability, and at large Rayleigh numbers Ra -- convective instability. In the general case in the space of the parameters $(\varepsilon Re, Ra, \lambda)$ we may find the surface of the neutral stability and determine the region of motion instability.

The system of equations for small perturbations may be greatly complicated as compared with the case of a fixed layer (1.7):

$$\left[\underline{L}^2(\underline{\Delta}-\sigma)+2Re\frac{\partial}{\partial\varphi}\right]w-2Re\,Q\cdot s-\bar{r}\,\mathrm{rot}\left[(\bar{u}\nabla)\bar{U}+(\bar{U}\nabla)\bar{u}\right]=0,$$

$$\left[L^2(\Delta-\sigma)+2Re\frac{\partial}{\partial\varphi}\right]\Delta s+2Re\,Q\cdot w-Ra\,\gamma(r)L^2\theta+$$

$$+\bar{r}\,\mathrm{rot}^2\left[(\bar{u}\nabla)\bar{U}+(\bar{U}\nabla)\bar{u}\right]=0,$$

$$Pr\,\sigma\,\theta+Pr\,\bar{U}\nabla\theta+\bar{u}\nabla T-\Delta\theta=0, \qquad (2.3)$$

where

$$Q=r\cos\vartheta\,\nabla^2-\left(L^2+r\frac{\partial}{\partial r}\right)\left(\cos\vartheta\frac{\partial}{\partial r}-\frac{\sin\vartheta}{r}\frac{\partial}{\partial\vartheta}\right).$$

The boundary conditions remain the same (1.8). The dependence of the Coriolis force components on the angle ϑ in a spherical rotating layer does not allow separation of the variables in the system (2.3). However, if we are interested in convective stability of the flow in a slowly rotating layer for a small latitudinal temperature gradient at the external boundary, then this difficulty /15 may be avoided by solving the system (2.3) with the boundary conditions (1.8) by the method of regular perturbations.

The basic flow becomes unstable at a certain critical value of the Rayleigh number, which in the general case must depend on other dimensionless parameters: $Re, Pr, \varepsilon, \lambda, r_0$.

If the characteristics of the basic motion and the perturbations are represented in the form of series with whole positive powers of Re and λ [11]

$$f = f_0 + Re f_{10} + Re^2 f_{20} + \dots + \lambda (f_{01} + Re f_{11} + \dots) + \dots, \quad (2.4)$$

then the solution of the boundary value problem (2.3), (1.8) reduces to solving a sequence of systems of equations with boundary conditions of the type (1.8). The system of equations for a zero approximation coincides with the system (1.7) for a quiet layer. Similarly, (2.4) may be represented in the form of the series Ra_c and σ), since the principle of stability displacement in rotating liquids is not valid in the general case and σ differs from zero. We should note that the rotation, shear and latitudinal gradient, as will be seen below, remove the degeneracy, since the critical number Ra_c depends on the azimuthal wave number m and its minimization makes it possible to establish the form of the perturbations precisely which arise at the stability limit.

It is apparent that the solution depends on the relationship between the parameters Re and λ. Therefore, we set $Re \sim \lambda^{\alpha}$. Then in different ranges of changes in the parameter α, other physical factors will have the basic influence upon flow stabilization and the convection which is produced.

In the case of a thin layer $r_0 \gg 1$ and the free boundaries, the solution of the problem is obtained in analytical form for arbitrary values of α. The limited nature of the viscous stresses in a thin layer requires that $\varepsilon = \varepsilon_0 r_0^{-1}$, where $\varepsilon_0 \lesssim 1$. /16

It was shown in [11] that in the following intervals in which α changes

1. $0 < \alpha < 1/2$, $\sigma \approx Re\,\sigma_{10}$, $Ra_c = Ra_0 + Re^{-}Ra_{20}$,

2. $\alpha = 1/2$, $\sigma \approx Re\,\sigma_{10}$, $Ra_c = Ra_0$,

3. $\alpha > 1/2$, $\sigma = Re\,\sigma_{10}$,

qualitatively different solutions are obtained; Ra_0 -- critical Rayleigh number for a spherical layer at rest. In the expansion of σ and Ra_c, only the main terms of the series are written which, in the given interval of change for α, take into account the contribution of the basic factor.

At $0 < \alpha < 1/2$ the stability limit, the form and the phase propagation velocity of the critical perturbations w are determined by the rotation and the shear. Outside of the region of values for the parameters ε_0 and Pr, lying between the curves ε_0^+ and ε_0^- (Fig. 1, the convective flow has the form of axisymmetric stationary rolls (m = 0). Between the curves ε_0^+ and ε_0^- the convection is three-dimensional, has the form of banana-like cells with an azimuthal wave number $m = \ell_c$, and is propagated in a wave-like manner in the φ direction with the phase velocity $\omega_0 = \varepsilon_0 Re/2r_0$, determined by the shear parameter ε_0. On the curves $\varepsilon_0^{\pm}(Pr)$ the convection form is indeterminate, and it is necessary to consider the following expansion terms with respect to Re in order to eliminate the degeneracy.

At $\varepsilon_0 > 0$ the critical Rayleigh number Ra_c is always greater than Ra_0, i.e., the shear flow in the rotation direction always stabilizes it independent of the form of the convection. A very large negative flow shear $\varepsilon_0 < -A_0/B \approx -3.5$ destabilizes the convection, although the form of the convection does not change. The dependence of $Ra_{20} = Ra_c - Ra_0$ on the Rossby number ε_0 is given in Fig. 2.

The value $\varepsilon_0 = 0$ corresponds to convection in a slowly rotating but solid layer and was studied in detail in [14]. In this /17
case, as may be seen from Fig. 1, convection is established in the

form of banana-like cells, but its velocity $\omega_1 = -\frac{Pr+1}{r_0^2} Re$ is always directed toward the inverse rotation of the layer, and is much less than w_0, since it is proportional to r_0^{-2}. We should note that in thin layers (greater than l_c) the axisymmetric stationary convection ($m = 0$) has maximum amplitudes in the polar regions, and on the other hand the banana-like convection ($m = l_c$) is concentrated close to the equator.

Allowance for the nonlinearity in this problem by studying the effects of second order terms of flow amplitude A shows that the redistribution of the angular convection moment leads in the second approximation to differential rotation of the layer. Actually, for the φ-components of the velocity field averaged over φ and r, the following analytical expression was obtained in [14]

$$\langle u_\varphi \rangle_{\varphi,r} = A^2 Re \frac{\ell}{6} (P_\ell^\ell)^2 \sin\vartheta \qquad (2.5)$$

for the case of a thin layer and free boundaries. Numerical calculations carried out for other boundary conditions show that the case of free boundaries to a certain degree is exclusive, since the value of the rotation greatly decreases for more general boundary conditions. However, in all cases the differential rotation is characterized by the equatorial acceleration of the layer.

This model for the interaction of convection with rotation may be used to explain the observational data regarding the Sun. If the theoretically examined convective cells are identified with gigantic cells on the Sun, and the action of the convective flows of smaller scales is taken into account by the introduction of the effective viscosity, then the amplitude of the differential rotation in the case of free upper and solid lower boundaries according to Busse estimates [14] coincides with the observed equatorial acceleration of the Sun. /18

At $\alpha = 1/2$, the total effect of shear, rotation and the latitudinal temperature gradient determines the parameters of critical convection. In Fig. 3, in the space of the parameters $\varepsilon_o, Pr, \lambda/Re^2$, the surface Σ given by the equation

$$C(Pr)\varepsilon_o^2 - B\varepsilon_o - A_o + A_1\lambda/Re^2 = 0$$

at $A_o = 8{,}985$, $B = 2{,}492$, $C(Pr) = 0{,}206Pr^2 + 0{,}289Pr + 0.37$
$A_1 = 3/4\, Ra_o$
delineates, together with the planes $\lambda/Re = 0$ and $Pr = 0$, the region within which the critical perturbations are realized in the form of three-dimensional banana-like cells with $m = \ell$, and outside of it in the form of the axisymmetric rolls with m = 0. The region of stabilization and destabilization of the basic flow is divided by the plane Π given by the equation

$$A_o + B\varepsilon_o - 1/3\, A_1\lambda/Re^2 = 0.$$

Below this curve there is always stabilization of the basic flow; above it -- destabilization. Axisymmetric convection is always stationary, and three-dimensional cells are propagated in a wave-like manner with the azimuthal phase velocity w_o.

When $\alpha > 1/2$, the stability limit and the form of the critical perturbations are determined by the latitudinal temperature gradient, and the decrement σ and the phase velocity w_1 proportional to it -- by rotation. Depending on the sign of λ , the minimum critical number Ra_c will be achieved in the case of axisymmetric convection, if $\lambda > 0$ and three-dimensional cells if $\lambda < 0$. In both cases, $Ra_c < Ra_o$, i.e., the nonuniform temperature distribution at the external boundary as compared with the uniform distribution leads to destabilization of the basic mode for one and the same total supply of heat passing to the layer [15]. /19

Thus, the total action on convection of all three effects in thin spherical layers is determined by the values of the similarity parameters ε_o, Pr and λ/Re^2. In the case of slow rotation (small Re), critical convection is always three-dimensional and has a wave nature [14]. The shear and the latitudinal temperature gradient at $\lambda > 0$ and in rotating layers contribute to the development of convective cells in the form of axisymmetric rolls. As applied to the astrophysical problem being considered, this means that allowance for solar radiation in a certain form may lead to a visible band-like structure of the planet surface. Actually, in adjacent rolls (two rolls form a convective cell), the gas in the meridional plane moves in opposite directions, forming ascending (warmer) and descending (cooler) streams. If, as is assumed, we interpret the light bands (zones) as ascending streams in the atmosphere of Jupiter, and the dark bands (belts) as descending streams, then the picture obtained for axisymmetric convection qualitatively explains the visible band-like structure of the surface.

If we assume that the band-like structure of the surface of Jupiter represents the occurrence of convective motions arising under the influence of an internal heat source and solar radiation, in the visible surface layer of the planet, and that this convection is close to a critical value with respect to the effective transport coefficients, then on the basis of the results obtained we may determine the thickness of the convective layer and the effective transport coefficients which are on the order of 150 km and 10^6 - 10^7 cm^2/sec, respectively [11].

Section 3. Influence of rotation on critical convection in the case of arbitrary Taylor numbers

In the case of the rotation of a spherical layer with an arbitrary velocity, the problem is greatly complicated, and complete separation of the variables cannot be achieved. It is possible to separate only the dependence in terms of longitude represented by the azimuthal wave number m. To study the influence of rotation on critical convection, it is necessary to solve a linear system of partial differential equations. /20

Gilman [16] obtained more complete results for this problem for a layer with the thickness $\delta = 0.25$ with free and infinitely heat-conducting boundaries. The calculations were performed for the Prandtl number Pr = 1. The system of linear equations in this study was solved by the method of finite differences in the (r, ϑ) plane for given m from 0 to 24. A determination was made of the rate at which the perturbations increased σ and the frequency of oscillations of the most unstable symmetric and anti-symmetric perturbations with respect to the equator for a given m. The Ra_c numbers at the stability limit for each m and the structure of the perturbations were found by interpolation up to the zero rate of increase. The Taylor numbers $Ta = 4\Omega^2 h^2/\nu^2 = 4Re^2$ changed from 0 to 10^6. The calculation results were the following:

1) The rotation had the strongest stabilizing influence upon modes which were anti-symmetric with respect to the equator, and on modes with low azimuthal wave numbers m (Fig. 4 and 4a);

2) With an increase in Ta, the wave number m_c of the most unstable perturbations increases, i.e., critical convection in a rotating spherical layer is always three-dimensional, and the dimensions of the cell in the azimuthal direction decrease with an increase in the rotation rate (Fig. 4);

3) With an increase in the rotation rate, the modes corresponding to different wave numbers m may be divided into two classes: a large part of the modes with high m have a maximum of amplitude on or close to the equator, and a small part with small m have a maximum at the pole (Fig. 5); /21

4) The equatorial modes of both symmetries are distributed in the azimuthal direction toward a side which is inverse to the rotation ($w < 0$), for a slow rotation of the layer in accordance with the results of Busse [14] and the side of rotation ($w > 0$) for large Ta (Fig. 6). Polar modes (with the exception of the stationary mode $m = 0$) are propagated at much slower rates;

5) The critical convective modes for large Ta are characterized by the asymptotic relations $Ra_c \sim Ta^{2/3}$, $m_c \sim Ta^{1/6}$, $\omega_c \sim Ta^{1/3}$, obtained earlier by Roberts [17] and Busse [32] for convection in rapidly rotating spheres;

6) In rapidly rotating layers, symmetric equatorial modes assume the form of rolls twisted around an axis parallel to the axis of rotation. The antisymmetric modes also have the form of rolls, but the direction of motion in them has different signs in different hemispheres and the liquid is input through the equatorial plane parallel to the axis of rotation. With an increase in Ta, the rolls are pressed into the internal sphere, obeying the Taylor-Praudmen theorem. The polar modes turn in a circular vortex, pressing closely against the poles;

7) Determinations show that the radial heat flux has a maximum at the equator for symmetrical equatorial modes, and at lower latitudes for antisymmetric modes. However, at large Ta, both fluxes are greatly suppressed close to the equator at the upper boundary of the layer. The symmetrical equatorial modes transfer the heat flux to the equator, and the antisymmetric ones -- to the

pole at the lower latitudes and to the equator at the higher latitudes. The symmetric equatorial modes transfer the angular moment to the equator from the high latitudes for al values of the Ta numbers;

8) For layers of other thicknesses $\delta = 0.11$ and 0.67, the behavior of the critical convective perturbations is similar to that described.

These results are confirmed by the studies of Roberts [17] and Busse [32] on the asymptotic modes of critical convection in spheres for very rapid rotation (Ta $\to \infty$, Eckman number E = $= \frac{\nu}{\Omega r_c^2} \ll 1$, r_c-- radius of the sphere). The dynamics of the convection which can be realized physically must in this case obey the Taylor-Praudmen theorem, according to which slow, steady motion of a slightly viscous liquid cannot depend on the coordinate along the rotation axis. Since such motions cannot be realized either in spheres or in spherical layers, the convective movements predominate over the limitations imposed by the theorem due to the nonstationarity and increase in the influence of viscosity due to a decrease in the scale of motion in the azimuthal direction. /22

At the stability limit, convection in the spheres arises close to a cylindrical surface located at a distance of approximately $1/2\ r_c$ from the rotation axis, and has the form of thin cylindrical rolls parallel to the rotation axis and slowly moving in the azimuthal direction at a velocity which depends on the Prandtl number. The horizontal scale of the rolls is approximately $E^{1/3}$ (Fig. 7).

It is clear from the form of the convection that the component of the gravitational force which is perpendicular to the rotation axis has the basic influence upon motion of this type.

Apparently in thick spherical layers with $h > \frac{r_c}{2}$ convection, just as in a sphere, arises close to the cylindrical surface with the radius $r_c/2$, and in thin layers with $h < r_c/2$ the convective rolls parallel to the rotation axis are tangent to the internal sphere, i.e., they have a maximum possible vertical scale and, consequently, a minimum deviation from the Taylor-Praudmen theorem.

Thus, at the stability limit in spherical layers both in the case of slow and rapid rotation, a non-axisymmetrical mode is established which is symmetrical with respect to the equator and with a high azimuthal wave number m. In the case of rapid rotation, the convection is concentrated in the pre-equatorial band, and the phase velocity is directed toward the side of the basic rotation rate. /23

Section 4. Nonlinear convection in a quiet spherical layer

The solutions of a linear system of equations which describe convection in a quiet spherical layer are, just as in the plane case, degenerate. This means that the solutions with a different spatial dependence (in this case with different azimuthal wave numbers m) correspond to one and the same eigenvalue -- the critical Rayleigh number Ra_c. Their realization at the stability limit is equally probable. This set of solutions is a reflection of the high degree of symmetry of the problem, but is this the physical result or is it due to disregarding important physical effects?

As is clear from the preceding, including even small additional effects (rotation, temperature nonuniformity, etc.) usually eliminates or decreases the degeneracy. One of the most important effects in the linear problem is the effect of the process nonlinearity. Therefore, it is important to establish whether allowance for nonlinear terms makes it possible to determine, in addition to the amplitude, the form of the finite-amplitude convection.

Close to the stability limit ($Ra - Ra_o$), the nonlinear terms may be regarded as a perturbation of the linear problem and the asymptotic method may be used for expanding the solution in powers of the small parameter -- the flow amplitude A. This method was used very successfully for studying the nonlinear convection motions Ra_o in the plane horizontal layer [18]. This method was used by Busse [19] to study the form of stationary finite-amplitude convective motions in a spherical layer. /24

The nonlinear system of equations

$$\Delta^2 L^2 s - Ra\,\gamma(r) L^2 \theta = F\,\nabla \times \{\nabla \times [\bar{u} \times (\nabla \times \bar{u})]\},$$
$$\Delta\theta + T'(r) r^{-1} L^2 s = Pr\,\bar{u}\,\nabla\theta \tag{4.1}$$

with allowance for the expansion in terms of amplitude

$$s = \sum_{i=1} A^i s_i\ ,\quad \theta = \sum_{i=1} A^i \theta_i\ ,\quad Ra = \sum_{i=0} Ra_i A^i \tag{4.2}$$

may be transformed into a sequence of linear nonhomogeneous systems of equations for S_1, θ_1 and Ra_1. The well-known system (1.9) is used as a system of the first order with respect to A. This system determines the critical convection in a spherical layer, the critical wave number l_c and the Rayleigh number Ra_o.

The general solution of the system may be given in the form

$$s_1 = S_1(r)\, v_l(\vartheta, \varphi);\quad \theta_1 = \Theta_1(r)\, v_l(\vartheta, \varphi),$$

$$v_l(\vartheta, \varphi) = \sum_{m=0}^{l} (\alpha_m \cos m\varphi + \beta_m \sin m\varphi) P_l^m(\cos\vartheta),$$

Here $2\ell+1$ coefficients α_m and β_m remain arbitrary due to the degeneracy of the system.

The system of second order with respect to A may be written as follows

$$\Delta^2 L^2 s_2 - Ra_0 \gamma(r) L^2 \theta_2 = -L^2[\nabla(\bar{r}\nabla+1)s_1 \nabla\Delta s_1 - \Delta s_1 \bar{r}\nabla\Delta s_1] + Ra_1 \gamma(r) L^2 \theta_1 ,$$

$$\Delta\theta_2 + \beta(r) r^{-1} L^2 s_2 = Pr[\nabla\times(\nabla\times\bar{r}s_1)]\nabla\theta_1 . \quad (4.4)$$ /25

The solvability condition (4.3) and (4.4) has the form

$$Ra_1 \langle \gamma s^+ L^2 \theta_1 \rangle - \langle s^+ L^2[\nabla(\bar{r}\nabla+1)s_1 \nabla\Delta s_1 - \Delta s_1 \bar{r}\nabla\Delta s_1] \rangle - Pr\, Ra_0 \langle \theta^+[\nabla(1+\bar{r}\nabla)s_1 - \bar{r}\Delta s_1]\nabla\theta_1 \rangle = 0, \quad (4.5)$$

where the brackets designate integration over the entire volume, and S^+ and θ^+ -- the solutions of a conjugate homogeneous sytem

$$\Delta^2 L^2 s^+ - Ra_0 \beta(r) r^{-1} L^2 \theta^+ = 0, \qquad \Delta\theta^+ + \gamma(r) L^2 s^+ = 0. \quad (4.6)$$

It is convenient to represent the solution (4.6) in the form

$$s^+ = S^+(r)\, v_\ell^+(\vartheta,\varphi) , \quad \theta^+ = \Theta^+(r)\, v_\ell^+(\vartheta,\varphi),$$

where $v_\ell^+(\vartheta,\varphi)$ -- spherical harmonics of order l. Then (4.5) may be written in the general form

$$Ra_1 \langle v_l^+ v_l \rangle - \langle v_l^+ v_l \; v_l \rangle M(l) = 0. \quad (4.7)$$

When the linear system corresponding to the nonlinear system (4.1) is self-conjugate, $M(l) \equiv 0$ for any 1. For nonlinear systems of the more general form, the condition of the linear system self-conjugate property does not lead to $M(l) \equiv 0$

We should note that the condition (4.7) is equivalent to the system $2l+1$ of nonlinear equations obtained due to the existence of $2l+1$ independent harmonics of the order 1.

The normalization condition v_e supplements the last missing equation for determining Ra_1

$$\langle v_e \, v_e \rangle = \alpha_0^2 + \sum_{m=1}^{l} (\alpha_m^2 + \beta_m^2) = 1. \quad (4.8)$$ /26

Since the system (4.7) - (4.8) is satisfied by the value $Ra_1 = 0$ for any α_m and β_m for odd 1, the solvability condition does not eliminate the degeneracy of the solution for odd 1. It may be readily shown that all Ra_n at 1 which are odd vanish. All even Ra_n are usually positive, and we may thus conclude that in the case of odd 1 the subcritical instability ($Ra < Ra_0$) may not occur in the spherical layer.

In the case of even 1, there is no general method for solving the system (4.7) - (4.8). Busse [19] examined several particular cases corresponding to 1 = 2, 4, 6 and very large even 1. A solution of the system (4.7) - (4.8) for any 1 is the axisymmetrical solution

$$\alpha_0^{(l)} = 1, \quad \alpha_m^{(l)} = \beta_m^{(l)} = 0, \quad m = 1, \ldots, l$$

with the corresponding value

$$Ra^* \equiv Ra_1/M(\ell) = (2\ell+1)^{3/2}\left[\left(\tfrac{3}{2}\ell\right)!\right]^2\left[\ell!\right]^3 \Big/ (3\ell+1)!\left[\left(\tfrac{1}{2}\ell\right)!\right]^6 \equiv k$$

In the case l = 2 the axisymmetric solution within an accuracy of the rotation transformation is unique and satisfies the condition of solvability.

In the general case of even l there are several solutions corresponding to different values of Ra_1, i.e., the degeneracy is only partially removed by the solvability condition. A certain physical principle is therefore necessary for selecting the flow which can be realized in practice. However, usually for selecting the finite-amplitude flow, its stability is studied. However, this is a very complex problem, and Busse in this study confined himself to the "softest" principle, namely he assumed that close to the stability limit the finite-amplitude flows may be realized physically with a minimum value of the Rayleigh number, i.e., the flows corresponding to the point E in Fig. 7a. /27

The minimum value of Ra is given by the formula

$$Ra_{min} = Ra_0 - \frac{Ra_1^2}{2\,Ra_2};$$

where Ra_2 is usually positive and reflects the change of the temperature field averaged horizontally due to convection, i.e., it changes very little with a change in l. If we disregard these changes and the contribution of the terms of higher order, it is apparent that Ra_{min} will correspond to the largest value of Ra_1.

At l = 4 and l = 6, there are other stationary solutions in addition to the axisymmetric solution. Thus, in the case l = 4 the value of R_a is reached for solving with cubic symmetry (Fig. 8), and at l = 6 -- for solving with dodecahedron symmetry (Fig. 9). For large even l, only partial solutions are obtained. However, the problem regarding the form of the most preferred mode remains open.

If the corresponding values of $Ra_1(l)$ are not very small and $Ra_0(l)$ has no sharply expressed minimum for odd l, then the realization of the convective flow with odd l is only slightly probable, and convective motions arise close to the stability limit in spherical layers as finite-amplitude subcritical instabilities corresponding to the even spherical harmonics. The form of these motions apparently has the greatest possible symmetry.

For a very large supercriticality, expansion with respect to amplitude is not applicable and it is necessary to solve the problem numerically. Durney [9] first attempted this, and calculated nonlinear convection in a spherical layer in the quasilinear approximation.

The basic assumption used in this approximation is disregarding the nonlinear terms corresponding to the intrinsic interaction of the perturbations, i.e., only those nonlinear terms remained in the equations which describe the interaction of the average temperature field with fluctuations of velocity and temperature [20]. /28

As Herring [20] showed, the quasilinear approximation gives good quantitative results when calculating convective flows with large Prandtl numbers. In the nonlinear case, this approximation permits separation of the variables. The finite-amplitude solution may be represented in the form of one unique mode, when the solution is decomposed in series with respect to the spherical harmonics $Y_l^m(\vartheta, \varphi)$. The system of equations of motion may be solved by the establishment method. Calculations for the layer with a thickness of $\delta = 0.25$ showed that convection at the stability limit and nonlinear motions in the case $Ra \leq 30 Ra_0$ are always stationary and correspond to the wave number $l = 10$, i.e., l equal to the critical value of the wave number for a given layer thickness.

The wave number realized in the case of convective motions may be determined by studying the stability of the solutions for given l with respect to small perturbations corresponding to $l-1$ and $l+1$ harmonics. For small supercriticality, the stable solution corresponded to the solution maximizing the heat flow, in accordance with the Malkus principle [21].

Certain types of convection (for example, flow with cubic symmetry, Fig. 8) obtained by Busse analytically for small amplitudes were calculated by Young numerically for convective motions of arbitrary amplitude.

Young [10] assumed a numerically complete nonlinear system of equations of hydrodynamics in the Boussinesq approximation for a spherical layer with free, infinitely heat-conducting boundaries. He used a rather original computational method: expansion in terms of spherical harmonics was used for angular variables, and the method of finite differences was applied with respect to the radius. The nonlinear terms were calculated using the modified transformation method of Orszag [22], and the order of magnitude of the other derivatives with respect to r in the system of equations was reduced using the Green function for the operator $\mathcal{D}_l$ and the one-dimensional diffusion equation. /29

The problem was solved by the establishment method for layers with the thickness $\delta = 2.33$ and 0.67, the Prandtl number Pr = 1 and 5 and the Rayleigh number $Ra \leq 5{,}5\ Ra_1$. Both axisymmetric and non-axisymmetric solutions were studied.

Basic results:

1) The axisymmetric solutions were not preferred for the complete three-dimensional equations of motion. There is an interval of values for the Rayleigh numbers $Ra_0 < Ra < Ra^*(Pr, \delta)$

where there are simultaneously two steady state modes depending on the initial data: the axisymmetric and the non-axisymmetric. A similar result for the dependence of the solution on the initial data was shown in the work by Schluter, et al. [18] for a plane horizontal layer. Krishnamurti confirmed experimentally [23] that three-dimensional cells in a plane layer may exist in the region of values for the Ra numbers where the two-dimensional rolls are stable.

2) At $Ra > Ra^*(Pr, \delta)$, the axisymmetric convection becomes unstable with respect to three-dimensional perturbations. The ratio Ra^*/Ra_0 decreases with an increase in the layer thickness δ and increases with an increase in the Prandtl number. Busse [24] obtained a qualitatively similar result pertaining to the instability of two-dimensional rolls with respect to three-dimensional perturbations in a plane layer.

3) In contrast to the quasilinear approximation [9], the dominating mode of the finite-amplitude solution does not always coincide with the most unstable mode at the stability limit. With /30 an increase in the Ra number, in individual cases there is a sharp change in the horizontal flow structure (decrease in the dominating wave number l), accompanied by a sharp change in the dependence of the heat flux on the Ra number (Fig. 10). This phenomenon has an analog in a plane layer and was observed experimentally in [23, 25, 26].

4) The dimensionless heat flux determined by the Nusselt number Nu increases with an increase in Ra/Ra_0 (Fig. 10). However, the difference in the heat fluxes transmitted in axisymmetric and non-axisymmetric modes is small, on the order of 5% (Fig. 11).

5) For those values of the parameters when there are stable stationary modes, an oscillatory three-dimensional convection mode may be established depending on the initial data. The dependence

of the oscillation amplitude on time has almost a sinusoidal nature, but the oscillation periods of the heat flux and the poloidal and toroidal mode are different.

It is important to note that toroidal modes in a quiet spherical layer are always negligibly small, except for the case of the oscillatory mode. This fact and the absence of oscillatory modes in the quasilinear approximation indicates that the occurrence of oscillations is related to nonlinear terms which describe the intrinsic interactions of the modes in the equations of motions.

The results obtained by Busse [19] and Young [10] are insufficient for formulating a general, although qualitative, representation of nonlinear convection in a quiet spherical layer. On the other hand, the studies of Busse [19] assume that close to the stability limit, just as in a plane layer when there is asymmetry between the upper and lower half of the layer, the three-dimensional cells are the preferred form of the finite-amplitude convective motions. In the opinion of Busse, the reason for this is the geometric asymmetry of the spherical layer. If we make an analogy with the plane layer later, then convection in the form of rolls must re- /31
place the three-dimensional structure, when the amplitude of the motion becomes large as compared with the asymmetry. However, the numerical calculations of Young point to the opposite picture: the axisymmetric convection becomes unstable with respect to three-dimensional perturbations with an increase in the Rayleigh number (at least for a Prandtl number which equals unity).

However, we must keep in mind that the calculations of Young pertain to Rayleigh numbers $Ra/Ra_0 \leq$ 1.3 - 1.4, whereas the results of Busse [19] are limited to small supercriticality. It is possible that the case $M(\ell) \equiv 0$, which was not studied by Busse corresponds to a plane layer which is symmetric with respect to the middle, and convection in this case is excited very smoothly and has the form of axisymmetric rolls.

But this pertains to the area of analogies. Many problems are still not explained. Is there an axisymmetric solution with a unique stable solution for small supercriticality? If the answer is yes, then in the space of the parameters (Pr, δ) where does the stability boundary lie of axisymmetric convection with respect to three-dimensional perturbations? What will occur in very thin layers $\delta \to 0$ (correct limiting transition to plane layer)? What is the influence of initial perturbations on the establishment of a certain form of convection motions? What is the influence of the Prandtl number on the transition from stationary convection to oscillatory convection?

Section 5. Nonlinear convection: interaction with rotation. Results of numerical calculations.

Calculations of the nonlinear convection in a rotating spherical layer were first performed by Durney [27] in the quasilinear approximation. This approximation in the case of allowance for rotation makes it possible to greatly simplify the problem due to the fact that modes with different azimuthal wave numbers m in the equations /32
do not interact directly. The case of azisymmetric convection $m = 0$ was examined for a layer with the thickness $\delta = 0.25$, the number $Ra = 1500$ and the Taylor number $Ta \leq 500$. The main influence of rotation on axisymmetric convection is reduced to its strong stabilization in the equatorial region, where pulsations of velocity, temperature, and the convective heat flux $H = \frac{1}{2} u_r \theta \sin\vartheta$ are suppressed (Fig. 12a).

As the calculation showed, an increase in the angular rotation velocity requires considering a large number of harmonics for an adequate description of convection. Thus, in the case $Ta = 500$, close to the value Ta for which convection is completely suppressed, it is necessary to consider modes with wave numbers from the interval $20 \leq \ell \leq 6$.

However, it is known that the strongest stabilizing influence of rotation is exerted on axisymmetric convection [28, 29]. Therefore, it is important to examine the non-axisymmetric case $m \neq 0$, which was done by Durney [30] for the number Ra = 1500 and the number Ta = 4.

To establish the preferred form of convective motion, integration was performed over time of a system of equations in which several modes were retained corresponding to different m. With time, all of the modes were damped except m = 10. In the equations, modes were retained with poloidal wave numbers l = 8, 10, 12 for temperature and the poloidal field component and l = 9, 11 for the toroidal component. We should recall that l = 10 is the critical wave number for a spherical layer at rest with the thickness $\delta = 0.25$ and the mode $l_c = 10$ is the unique stable mode when calculating nonlinear convection in a spherical layer at rest in the quasilinear approximation at Ra = 1500.

Thus, for small Taylor numbers, the convective flow arises in the form of three-dimensional cells with $m = l_c$, which coincides with the results of Busse for critical convection in thin layers [14]. The dependence of the solution on time has, in accordance with [14], a specific form $\exp i(m\varphi + \sigma t)$, which is characteristic for gravitational-hygroscopic waves. The value of σ does not depend on r and ϑ, and in this case equals 3.755. /33

The convective heat flux $\langle u_r \theta \rangle$, averaged over φ, does not depend on time and is only a function of r and ϑ. Figure 12b shows the dependence $\langle u_r \theta \rangle$ on ϑ, in the middle of the layer. It may be seen that the heat flux is maximal at the equator, i.e., non-axisymmetric convection is more effective at the equator, in contrast to axisymmetric convection.

After the solutions are found in the quasilinear approximation, we may calculate the nonlinear terms corresponding to the intrinsic interaction of the modes and determine how the angular moment is redistributed by convective motion. This procedure is valid if the nonlinear terms are small as compared with terms which are considered in the quasilinear approximation. Figure 13 shows the dimensionless part of the angular frequency depending on ϑ, at the external boundary as a function of the angle ϑ.

Thus, although the convective motions are non-axisymmetric, the intrinsic interaction of non-axisymmetric modes in the subsequent approximation excites the axisymmetric mode, leading to differential rotation of the layer and equatorial acceleration.

This property of convective motion causes differential rotation at the external boundary and may be used to explain the equatorial acceleration of the Sun [30].

The studies of Gilman [16] and Busse [14] of critical convec- /34
tion in a rotating spherical layer and of Young [10] and Busse [19] on nonlinear convection in a quiet layer showed that both effects (rotation and ronlinearity) operate in one direction, i.e., they cause non-axisymmetric convection. Durney came to the same result [30], when studying the interaction of nonlinear convection with rotation in a quasilinear approximation.

Therefore, at first glance the result obtained in the study of Williams and Robinson [31] is somewhat unexpected, where a numerical calculation was made of nonlinear convective motion in the approximation of a "semi-thin layer" for the $Ra \leq 9502 \approx 9 Ra_0$ number and the Taylor number $0 \leq Ta \leq 5 \cdot 10^5$ and $Pr = 7.1$ (Ra_0 -- critical Rayleigh number in a quiet layer). The calculations employed the grid method, the equations were integrated over time for a layer with the thickness $\delta = 0.15$ with a free upper and rigid lower boundary, and both boundaries were assumed to be infinitely conducting.

The basic result is reduced to the fact that in this range of

Ra and Ta numbers, a stationary axisymmetric convective mode is established which is stable with respect to three-dimensional (azimuthal) perturbations.

It may be assumed that this divergence in the results may be explained by the large value of the Prandtl number used in [31]. However, this is not the case and may be seen from calculating A-12 (see [31], table). Actually, this case Ra = 2376 and Ta = $2.9 \cdot 10^5$ pertains to convection at the stability limit (Nusselt number Nu = 1.001), and as is known, the Prandtl number is not included in linear equations in the steady state case. The decisive factor is the fact that the authors used the "semi-thin" layer approximation, in which they disregarded the components of the Coriolis force, proportional to $\sin\vartheta$ and making the greatest contribution at the equator (i.e., the r-component and the part /35
$\Omega_c u \sin\vartheta$ -- the azimuthal component of the Coriolis force).

The authors explain the difference with the results of calculating critical convection in a thin layer [14] by the fact that the analysis of Busse applied only to slow rotation.

The behavior of the axisymmetric solution [31] differed greatly from the axisymmetric flows of Gilman [16] and Durney [27]. In contrast to the latter, the axisymmetric flow in the approximation of a "semi-thin" layer was suppressed at the poles, and not at the equator with an increase in the angular velocity of rotation (Fig. 14).

As may be seen from the latitudinal velocity profiles (Fig. 14), for different Ra and Ta, the zonal flow in the solution which is symmetric with respect to the equator has a maximum close to the equator at the upper layer boundary. There is a small decrease in velocity at the equator itself.

The axisymmetric structure of convective flow, the acceleration of the flow close to the equator, the alternating ascending and descending flows in adjacent axisymmetric rolls make this model of convection very interesting for explaining the observed phenomena on the surface of Jupiter.

The convection calculation performed by the authors, taking into account the planetary scales, showed that equatorial acceleration corresponding to the observational data may be obtained for a comparatively fine atmosphere of approximately 150 - 500 km. However, it must be assumed that there is strong anisotropy in the transport coefficients in the horizontal and vertical directions $\nu_{\parallel}/\nu_{\perp} \sim 10^6$.

Section 6. Laboratory modeling of astrophysical convection.

The difficulties arising when creating a gravitational field with spherical symmetry under laboratory conditions lead to the fact that the experimental studies on convection are limited to a plane layer. However, convection in a plane layer has the property of horizontal isotropy, which does not exist in rotating spherical layers with intrinsic gravitation. Therefore, the analogy with experiment is very poor. The greatest difference is due to the fact that the angles between the force of gravitation and the angular velocity vector in plane and spherical cases do not coincide. /36

However, in the case of rapid rotation, as shown by Busse [33], due to the specific form of convection in the form of columns parallel to the axis of rotation, the basic force is the component of the force of gravity perpendicular to the angular velocity. This fact is the starting point for laboratory modeling of astrophysical convection performed in the division of Space Physics of Columbia University [34]. The force of gravitation was modeled in the experiments by centrifugal force arising during the rotation of a spherical layer around a vertical axis. Figure 15 shows a diagram of the laboratory equipment. To compensate for the difference in the signs of centrifugal and gravitational force under laboratory conditions, the direction of the temperature gradient was changed to the opposite direction. The spherical layer between the external (transparent) spherical shell and the internal sphere was filled with liquid, to which aluminum flakes were added to visualize the flow. The layer was placed in a transparent container, in which thermostatically controlled water circulated to maintain the temperature at the external boundary of the layer. To maintain a constant temperature of the internal sphere, the thermostatically controlled water was added along the axis of rotation at a lower temperature.

The preliminary results now published qualitatively describe certain convective modes which are produced. For a high rotation /37
rate and a corresponding ΔT, a very non-axisymmetric convective mode was established, having the form of vertical columns regularly distributed in space (Fig. 7a). With a further increase in the rotational velocity, the convective columns were filled with the external section of a spherical ring. Due to the difference in the phase propagation velocities, the location of the columns in space was irregular. However, their form was rigorously parallel to the rotation axis.

The thin layers make it possible to implement flows with relatively low Taylor numbers without decreasing the rotational velocity below the limit when the gravitational force begins. In this case convection assumes the form of banana-like cells obtained theoretically in slowly-rotating thin layers [14].

However, there is another possibility for modeling convection in spherical layers with central gravitation The basic idea is to produce a central field of forces by an electric field. It was tested on equipment with spherical geometry by Chandra and Smylie [35] at the Columbia University in Canada.

In this case the central field of forces was produced by the difference in the potentials V on external and internal spheres. Heating the internal sphere and cooling the external one established the necessary temperature gradient. The calculation showed that the volumetric force directed along the radius of a spherical layer has the value

$$f_e = \frac{1}{2\pi} \frac{V^2 r_i^2 r_e^2}{r^5 (r_e - r_i)^2} \frac{1}{\rho_0 \alpha} \left(\frac{\partial \varepsilon}{\partial T}\right)_p ,$$

where $\left(\frac{\partial \varepsilon}{\partial T}\right)_p$ - temperature coefficient of the dielectric con- /38
stant. For a thin layer $r_i \sim r_e$.

$$f_e = \frac{1}{2\pi} \frac{V^2}{r_i h^2} \frac{1}{\rho_0 \alpha} \left(\frac{\partial \varepsilon}{\partial T}\right)_p .$$

In dielectrics of the silicon oil type at T ~ 20° C

$$\left(\frac{\partial \varepsilon}{\partial T}\right)_p \approx 4 \cdot 10^{-3} \,{}^\circ C^{-1}, \quad \alpha \approx 10^{-3} \,{}^\circ C^{-1}$$

$$f_e \approx \frac{2}{\pi} \frac{V^2}{r_i h^2} .$$

In order that the force of the Earth attraction does not have a great influence upon the convection produced, the following is necessary:

$$f_e / g \gg 1.$$

If we set f_e / g = 10, then for a field strength of V/h = 100 kV/cm, the following condition is obtained for the geometry of the layer $r_i h$ = 7.3 cm, i.e., the dimensions of the equipment are very limited. The basic limitation is imposed by the strength of the field V/h. It must not exceed a certain value of ≈100 kV/cm, at which breakdown may occur in the liquid.

It is apparent that the use of the electric field for modeling convection in a spherical layer was implemented most effectively by Busse and Carrigan, but the equipment was more complex to use.

CONCLUSION

The problem of convection in a spherical layer without considering any complicating factors (rotation, shear, etc.) is determined by three-dimensionless parameters: the Rayleigh number Ra, the Prandtl number Pr, and the dimensionless thickness of the layer δ, i.e., one parameter of similarity more than in the problem of Raleigh-Benar.

Just as in the problem of convection in a plane layer, the linear problem is degenerate, but $2\ell_c+1$ solutions correspond to the critical Rayleigh number in a spherical layer, and not an infinite set, and ℓ_c greatly depends on the layer thickness and, to a lesser extent, on the boundary conditions. /39

Allowance for small supercriticality of the motion does not completely eliminate the degeneracy, although it greatly reduces the possible solutions [19]. Just as in the plane case, for one and the same values of the similarity parameters, there is nonuniqueness of the solutions for nonlinear convection. However, in contrast to the plane layer, the horizontal dimensions of finite-amplitude flow are completely determinate, and the arbitrary selection of a poloidal wave number is impossible. This is due to the closed nature of the volume. The nonuniqueness of the solution may be due to the form (azimuthal wave number) or nonstationary nature of the solutions.

The existence of such complicating factors as rotation, latitudinal temperature gradient at the boundary, shear flow leads to the elimination of degeneracy in the linear problem [11]. Shear and the latitudinal gradient with $\lambda > 0$ lead to establishing axisymmetric convection, and rotation causes three-dimensional cells having an intrinsic phase propagation velocity in the azimuthal direction. In thin layers, even in the case of slow rotation, convection is concentrated in the equatorial region.

At large rotation velocities $Ta \to \infty$, critical convection in the polar regions is suppressed, and convective cells assume the form of thin rolls -- columns parallel to the axis of rotation. The critical convective modes at $Ta \to \infty$ are characterized by the asymptotic relations:

$$Ra_c \sim Ta^{2/3}, \ m_c \sim Ta^{1/6}, \ \omega_c \sim Ta^{1/3}.$$

A study of the interaction between nonlinear convection and rotation has only begun. The studies of Durney [27, 30] and Williams and Robinson [31] pertained to specific approximations which cannot give a complete representation of nonlinear convective flows in spherically rotating layers. The difficulties consist of the fact that modes of nonlinear convection which can be physically realized (stable) in this case are always three-dimensional, which greatly complicates the use of numerical methods. /40

The studies performed on convection in spherically rotating layers show the promising use of the simple models for understanding and explaining the observational phenomena on planets and stars. These models help to explain the most characteristic aspects of the phenomenon, in the case of convection in rotating spherical layers. These models also provide a basis for understanding the mechanism of the phenomenon as a whole and the formulation of more complex models which are closer to real conditions, whose numerical modeling is still very difficult [33,36].

Along with this, certain difficulties have arisen. As was shown in [14, 30, 31], the interaction of rotation with convection leads to two basic effects: transport of the angular moment to the equator and stabilization of large-scale convection at the poles. A consequence of the second effect is the great difference in the heat fluxes and temperatures at the poles and the equator at the upper boundary of the convective zone; this difference was not observed either on the Sun [37] nor on Jupiter [38].

In addition, convection in rapidly rotating spherical shells strives to assume a non-axisymmetric form in the form of cells extended in the meridional direction, and this contradicts the observations on Jupiter.

Recent results of spectroscopic measurements on "Pioneer 10 /41
and 11" [38] and theoretical studies [33, 36] show that the convective model of Jupiter must be more complex than was assumed previously.

The model of Jupiter must explain the absence of a difference in the heat fluxes at the equator and at the poles and the development of convective cells in the form of axisymmetric rolls parallel to the equator. Studies we performed earlier [11, 36] lead to the assumption that both of these phenomena may be related to the absorption of solar energy close to the upper boundary of the convective zone.

Actually, it is difficult to assume that the heat flux from the Sun, which comprises approximately half of the internal heat flux, does not have a great influence upon the hydrodynamics of large-scale phenomena in the atmosphere of Jupiter. Apparently, in the upper layers of the atmosphere where primarily solar radiation is absorbed, the viscosity is small (the penetrating convection is suppressed due to the stable temperature stratification) and due to a geostrophic balance zonal westerly flows arise. The action of these flows on the underlying layers may be represented as tangential stresses $\tau_{z\varphi}$, which are continuously distributed at the boundary and which change with latitude. In the case of rapid rotation of the planet $Re \gg 1$ and for small Rossby numbers $\varepsilon \ll 1$, the influence of $\tau_{z\varphi}$ on the lower layers leads to the formation of a cylindrical shear layer parallel to the axis of rotation and tangent to the lower boundary of the convective zone at the equator. As was shown in [36], the extension of this layer to the external boundary causes a sharp change in the angular velocity at the surface, and may thus explain the existence of an equatorial stream in the region of $\pm 10^{\circ}$ of the Jupiter latitude.

In addition, a strong zonal flow at the upper boundary, together with the latitudinal temperature gradient [11], in the presence of rotation contribute to the development of convective /42
cells in the form of axisymmetric rolls. The band-like structure of the surface of Jupiter at latitudes of $\pm 10^{\circ} - \pm 45^{\circ}$ may be the result of large-scale convective cells of this type reaching the surface.

The difference in the absorption of solar radiation at the equator and the poles must lead to a decrease in the convection intensity in the pre-equatorial zone and may balance the influence of rotation causing a decrease in the intensity of convection at the poles. Actually, both of these factors, rotation and absorption of solar radiation, will suppress the convection selectively: rotation -- at the poles; absorption -- close to the equator. As a result, the heat flux produced by the planet at different latitudes may be approximately the same.

The three-dimensional structure of convection at high latitudes

[38] must be explained by a decrease in the velocity of the zonal flow and the absorption of solar energy [11]. It is apparent that the model approximates the real one most closely if convection is regarded in a compressed medium, in contrast to the Boussinesq approximation.

The situation is more complex with the solar convective model. Although the model of the convective shell has good agreement with the observational data referring to large-scale motions in the photosphere (differential rotation, dimensions of gigantic cells, weak meridional circulation, etc.), there is a great contrast between the observed and calculated heat fluxes on the surface.

Rough estimates of the difference in the heat fluxes at the equator and the poles ΔF for a deep model of the solar convective zone give values of $\Delta F/F \sim 15-20\%$ [39,40]. There is no validity to the ideas that the equations and boundary conditions are not sufficiently realistic for predicting heat fluxes close to the upper boundary of the convective zone, since the mechanism of /43 transfer from convective heat transport to radiant heat transport close to the boundaries was not considered. Additional concepts regarding the possible redistribution of heat flux within the convective zone are necessary.

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Fig. 12. Convective heat fluxes for the number Ra = 1500 in the middle of a layer with the thickness δ = 0.25 as a function of the angle ϑ:

a) axisymmetric mode M = 0, Ta = 500;

b) non-axisymmetric mode m = 10, Ta = 4.

Fig. 13. Dimensionless part of angular velocity depending on ϑ, at the external boundary of the layer.

Fig. 14. Latitudinal profiles of normalized zonal V and radial W velocity and temperature in the middle of the layer for different values of Ra and Ta.

A 0 - $Ra = 2376$, $Ta = 0$;

A 2 - $Ra = 2376$, $Ta = 2 \cdot 10^3$;

A 8 - $Ra = 2376$, $Ta = 1.3 \cdot 10^5$;

B 8 - $Ra = 4751$, $Ta = 1.3 \cdot 10^5$.

Fig. 15. Diagram of equipment for modeling the astrophysical convection.

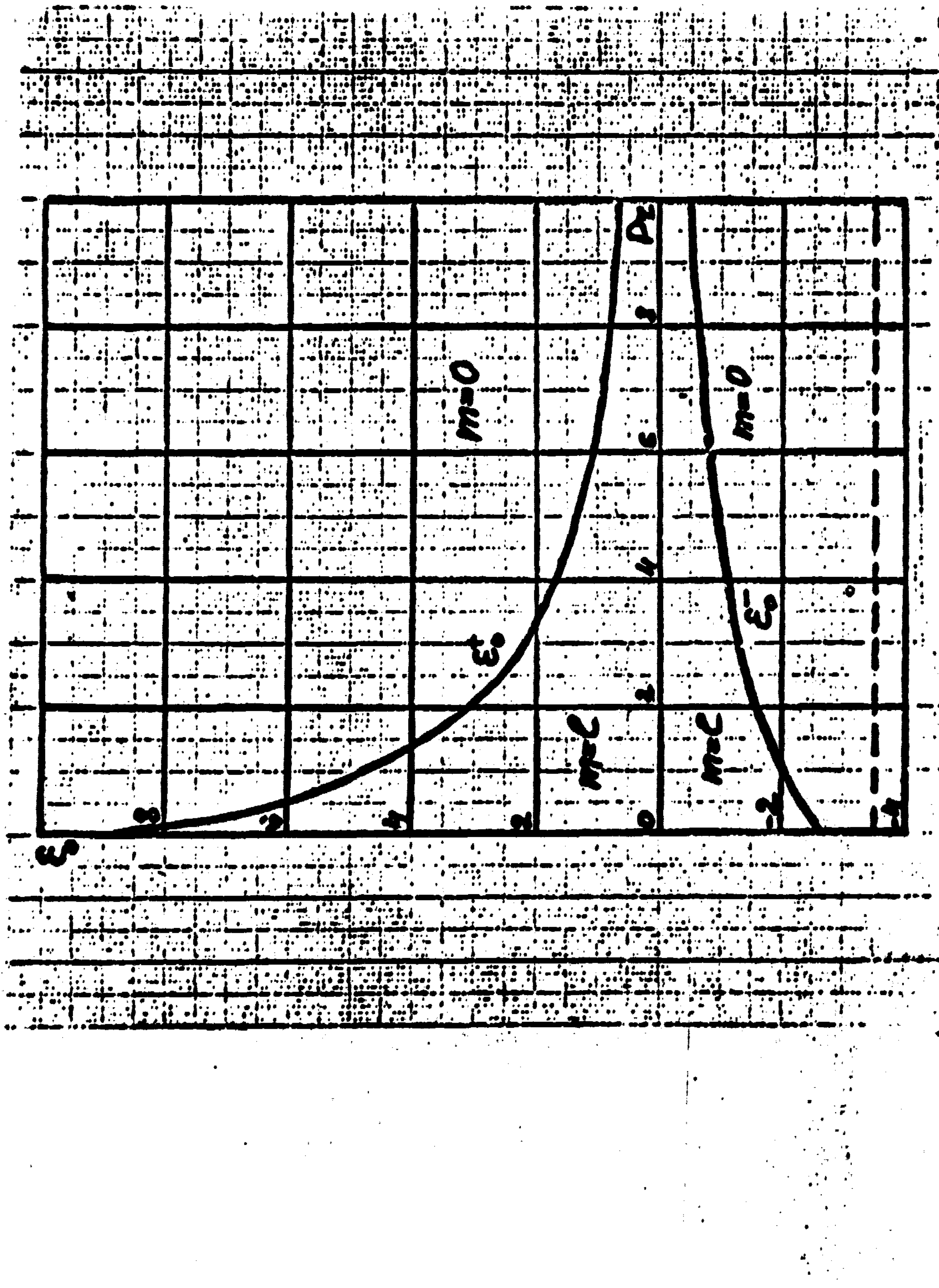

Fig. 1

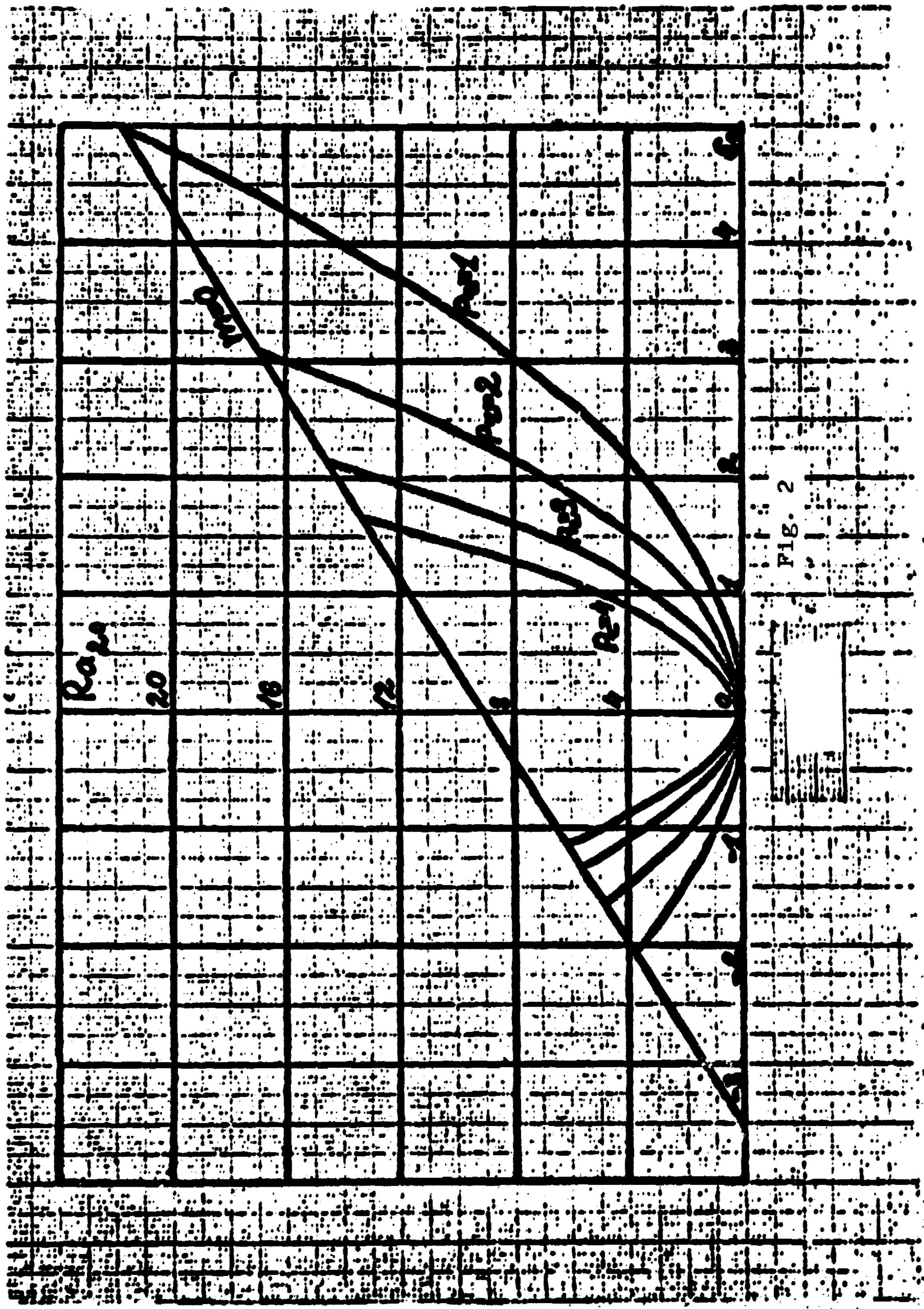

Fig. 2

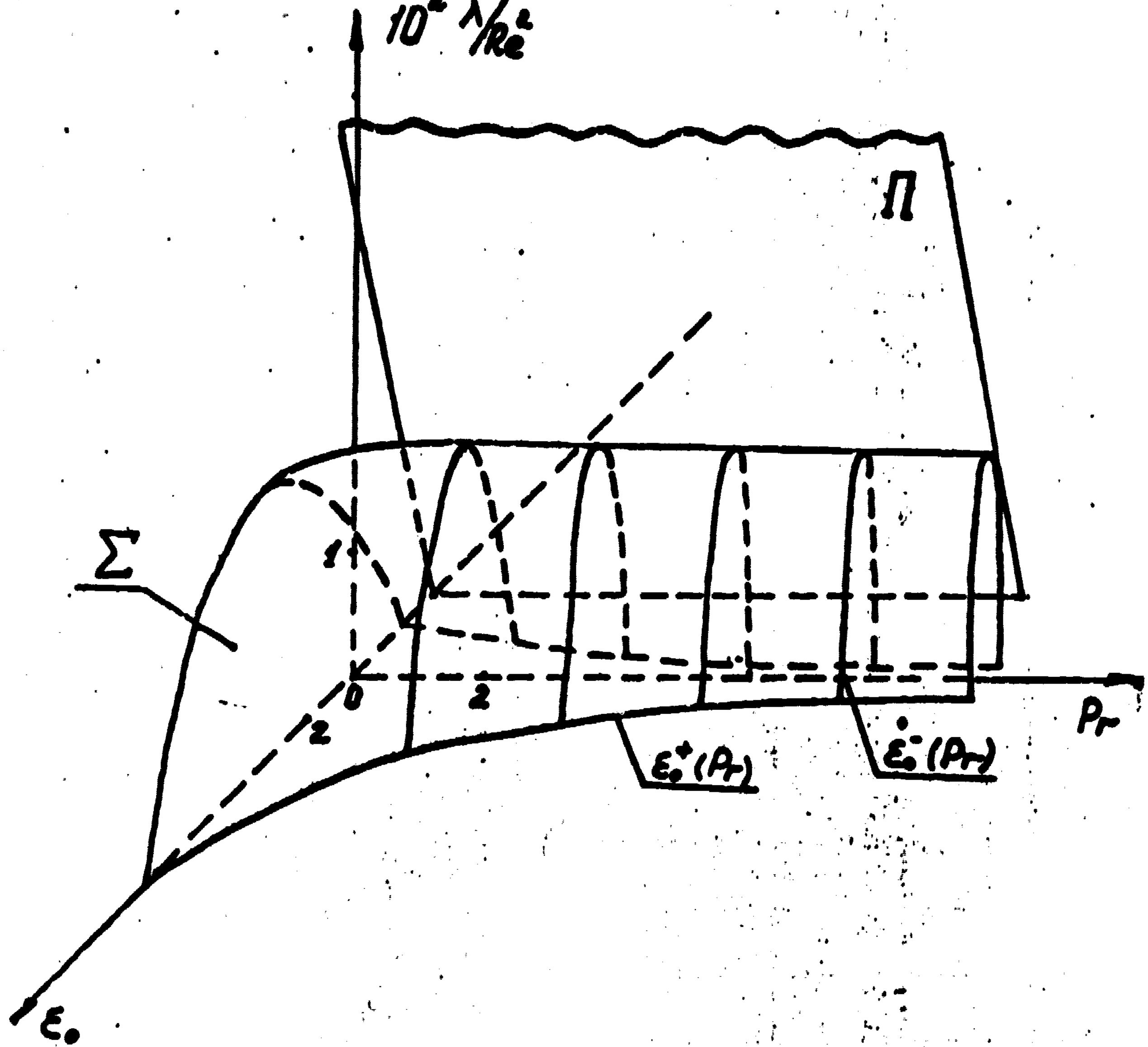
$10^2 \lambda/Re^2$
Π
Σ
1
0
2
2
P_r
$\varepsilon_0^+(P_r)$
$\dot{\varepsilon}_0^-(P_r)$
ε_0

Fig. 3

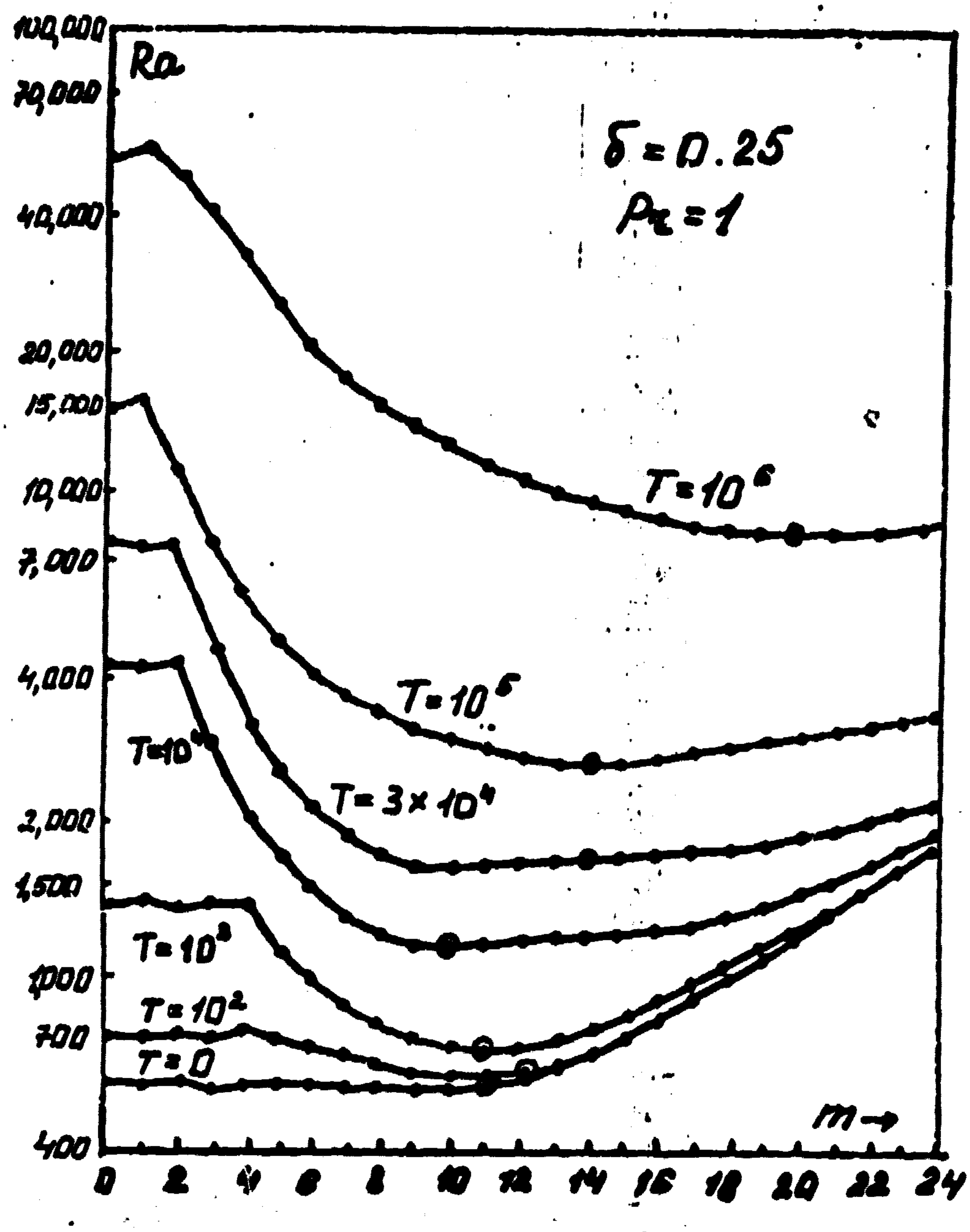
Ra
100,000
70,000
40,000
20,000
15,000
10,000
7,000
4,000
2,000
1,500
1000
700
400
δ = 0.25
Pr = 1
T = 10^6
T = 10^5
T = 3 × 10^4
T = 10^4
T = 10^3
T = 10^2
T = 0
m →
0
2
4
6
8
10
12
14
16
18
20
22
24

Fig. 4

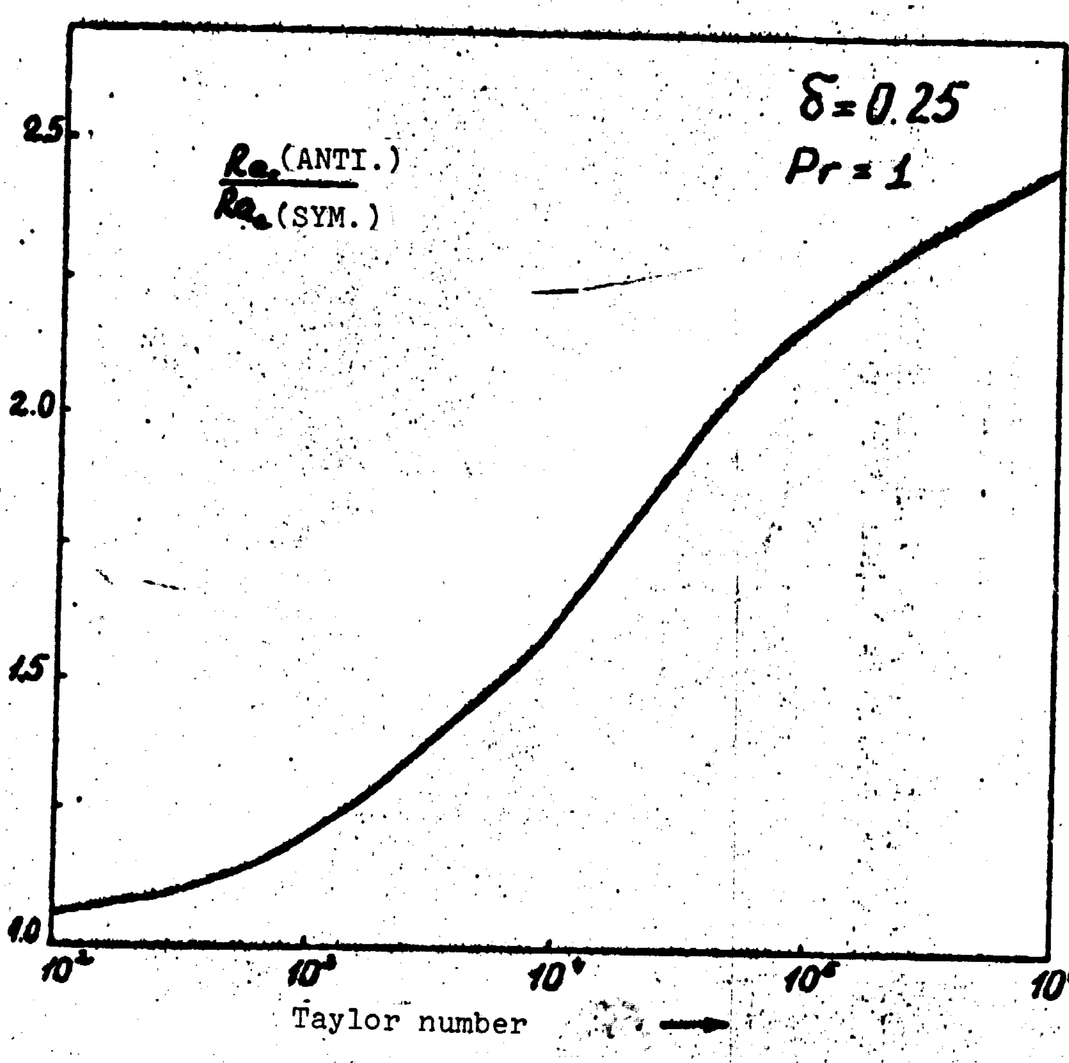
δ = 0.25
Pr = 1
Re_c (ANTI.) / Re_c (SYM.)
2.5
2.0
1.5
1.0
10^2
10^3
10^4
10^5
10^6
Taylor number

Fig. 4a

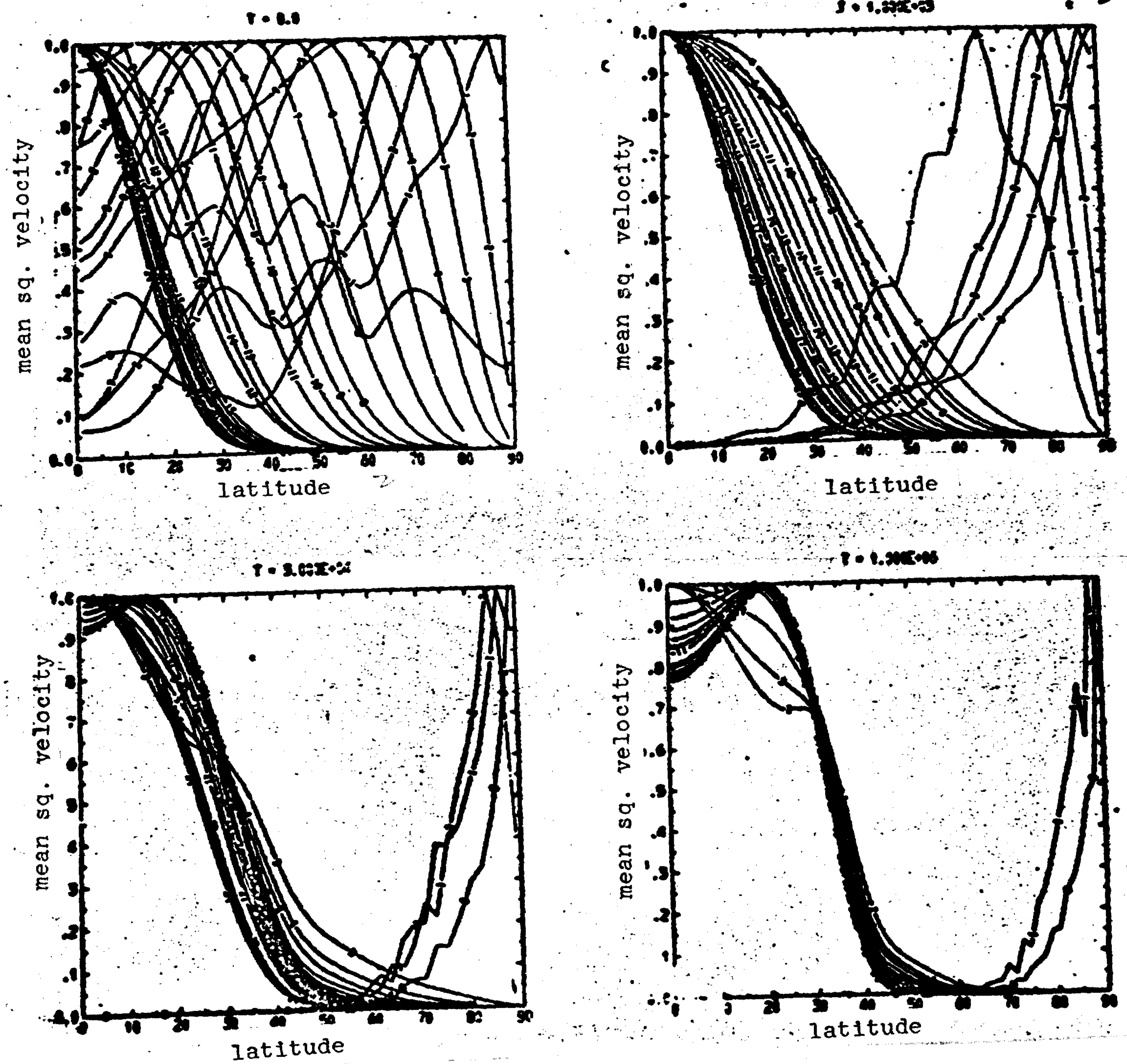
T = 0.0
mean sq. velocity
latitude
mean sq. velocity
latitude
mean sq. velocity
latitude
mean sq. velocity
latitude

Fig. 5

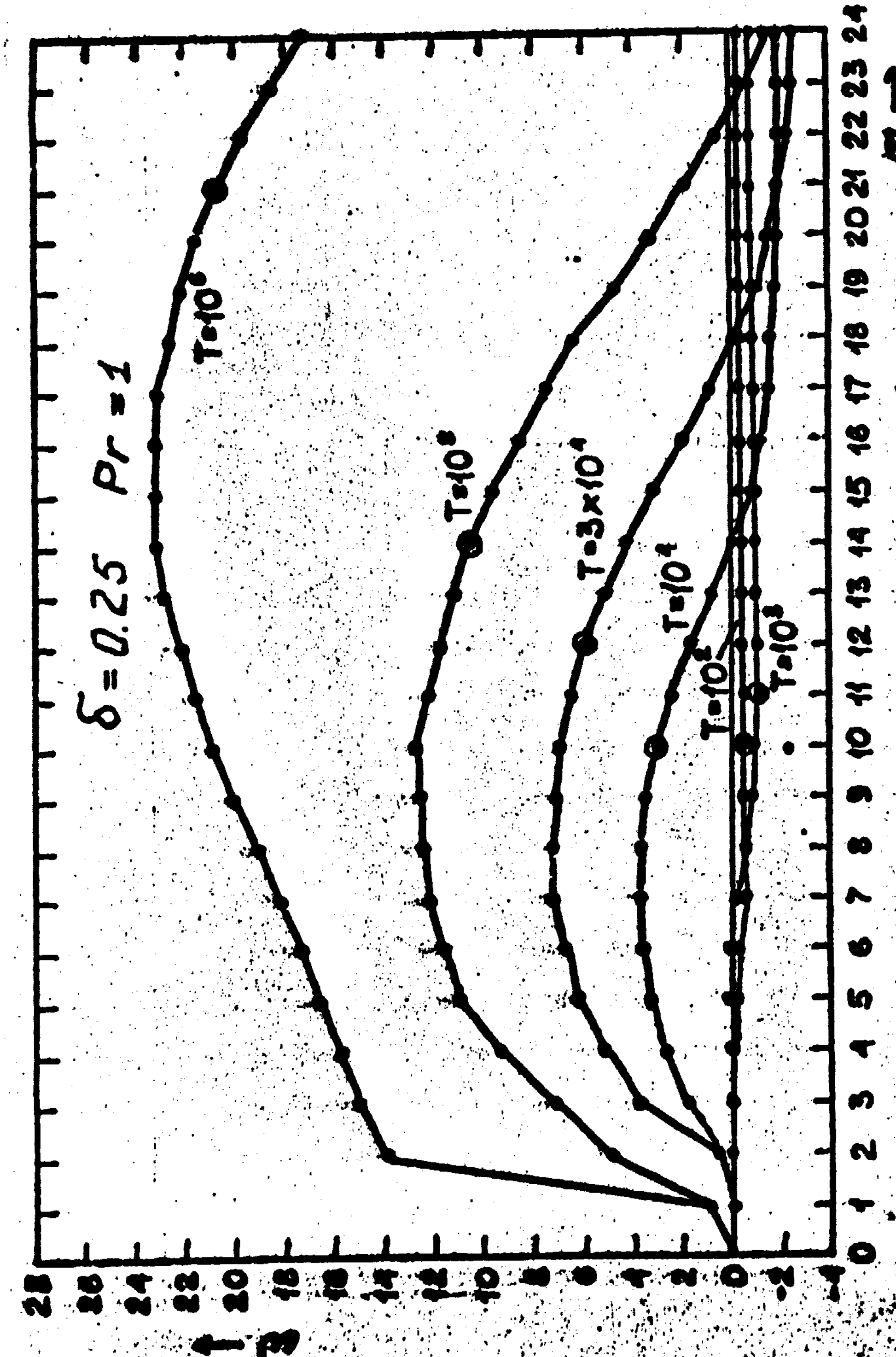
δ = 0.25 Pr = 1
T=10^6
T=10^5
T=3×10^4
T=10^4
T=10^3
T=10^2
m →
ω →
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
-4 -2 0 2 4 6 8 10 12 14 16 18 20 22 24 26 28

Fig. 6

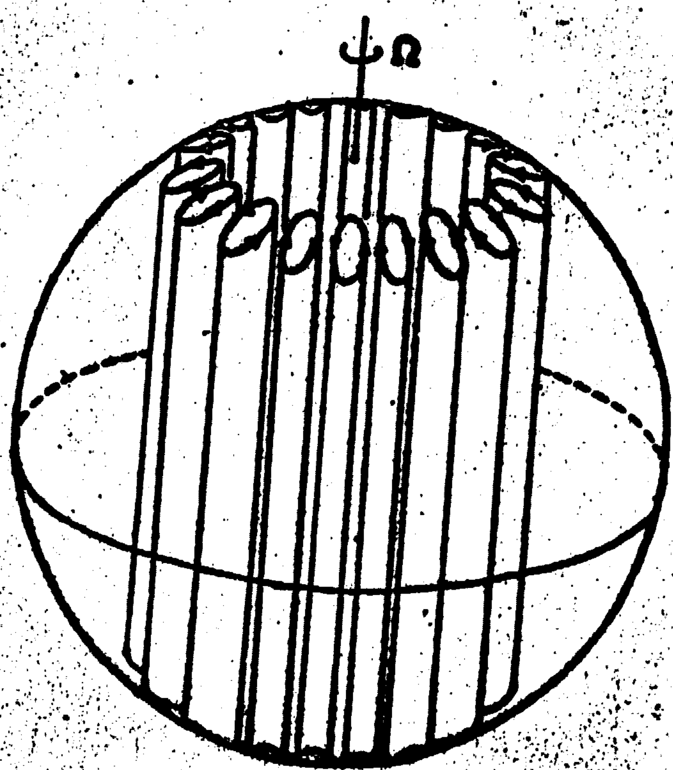

Fig. 7

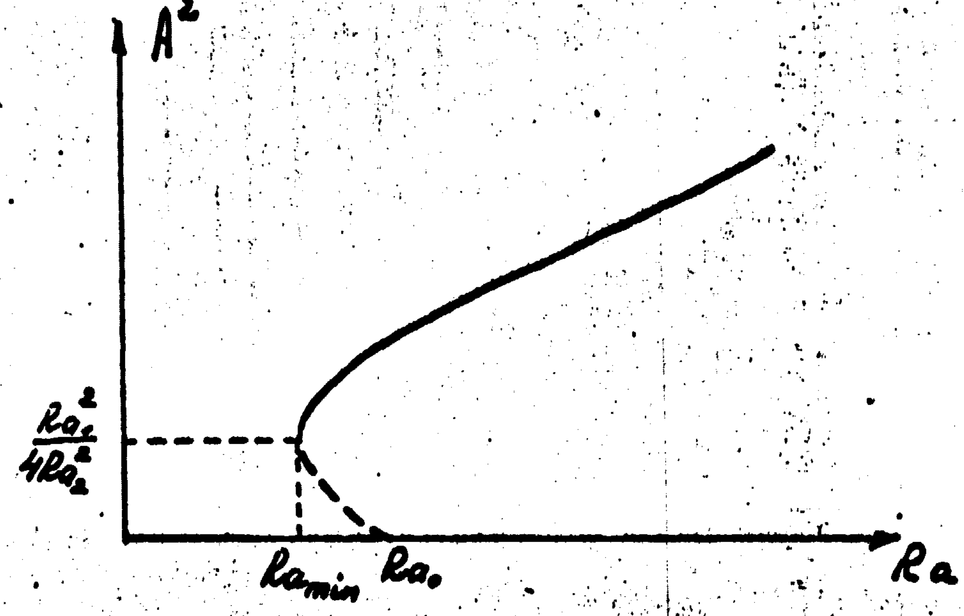

Fig. 7a

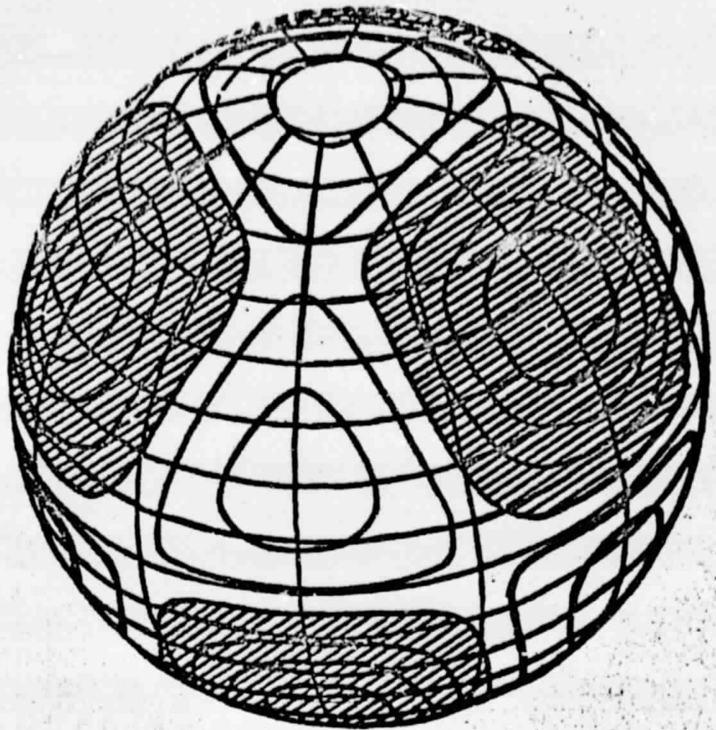

Fig. 8

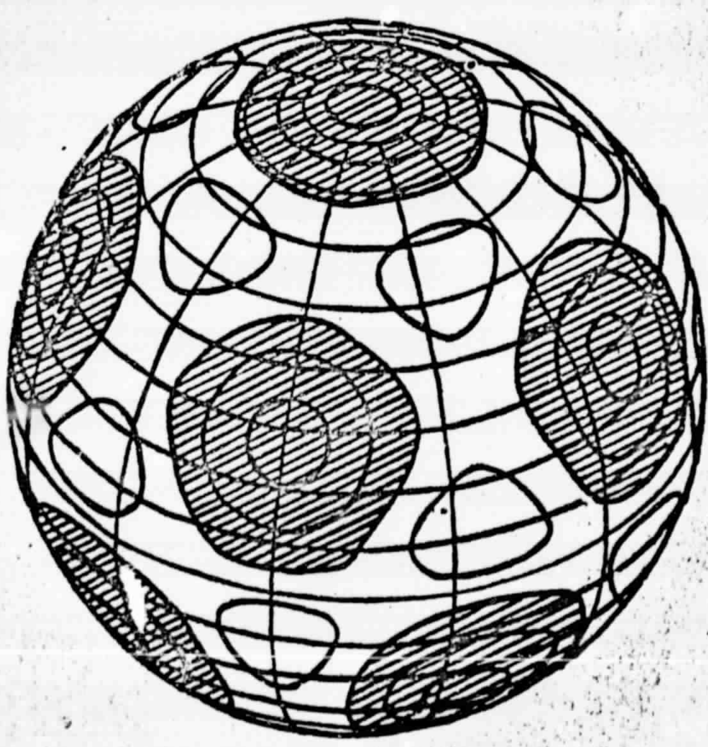

Fig. 9

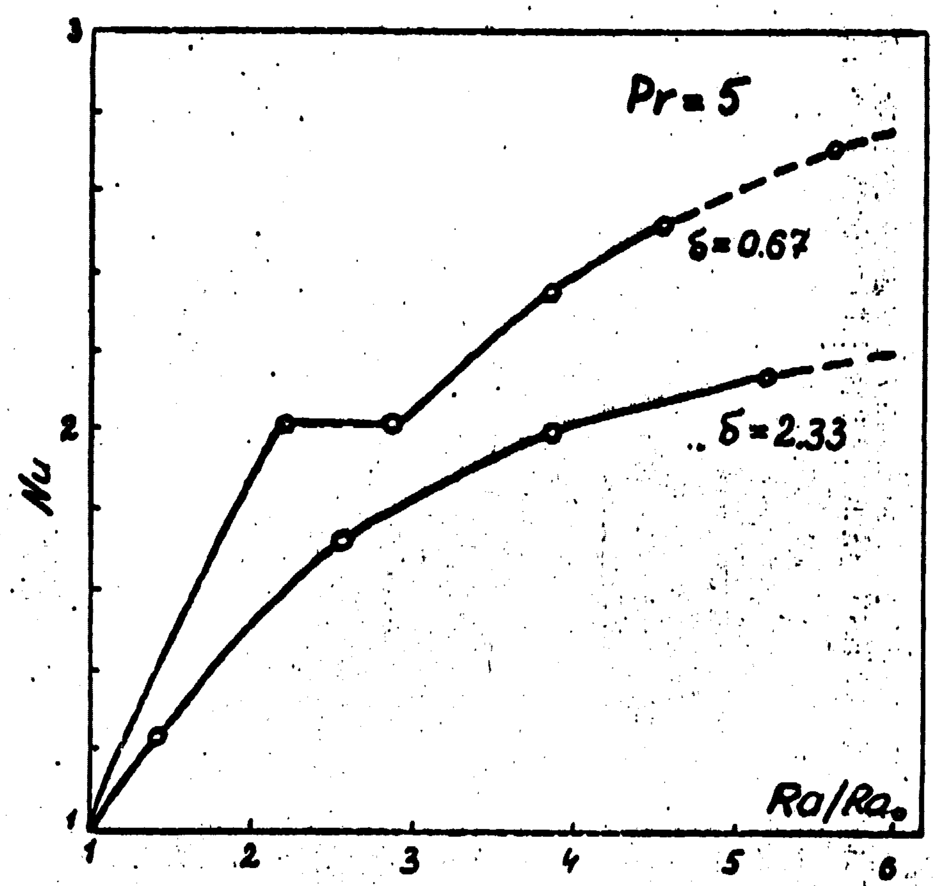

Fig. 10

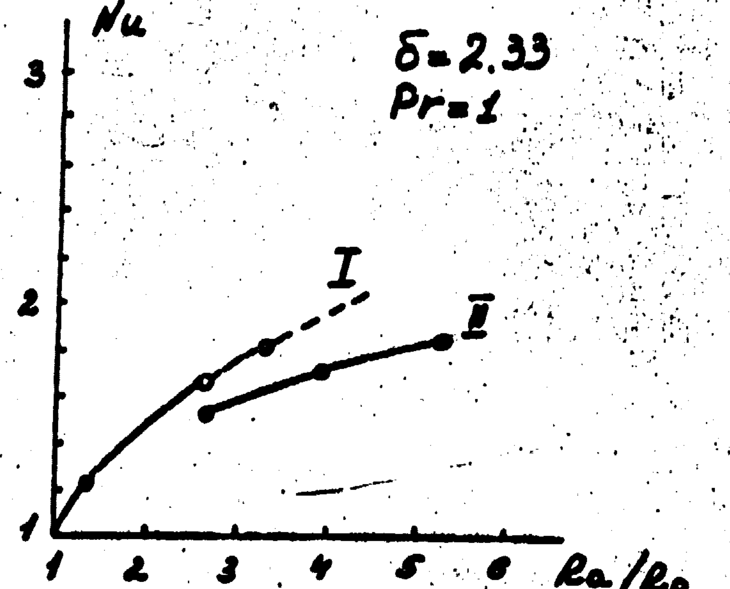

Fig. 11

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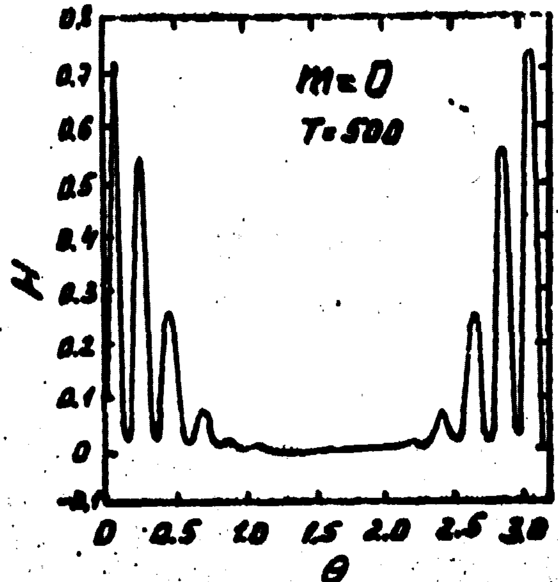

Fig. 12a

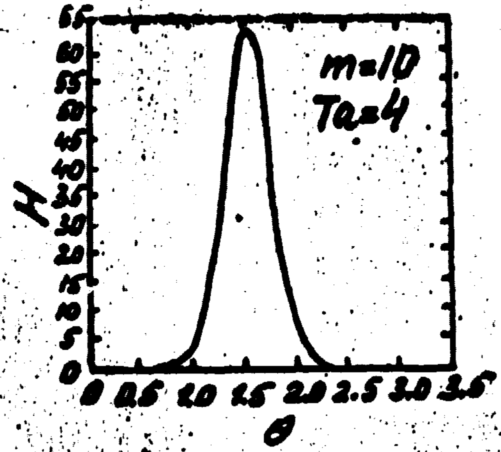

Fig. 12b

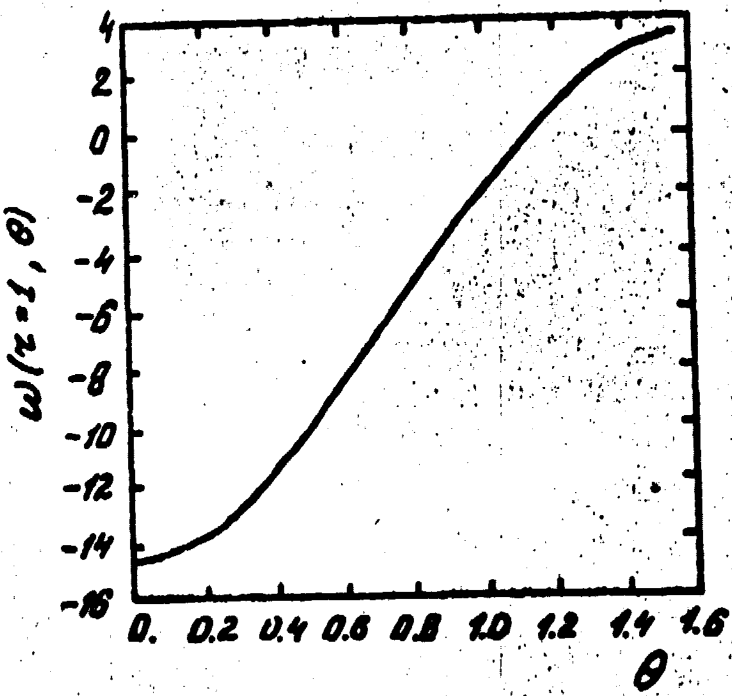

Fig. 13

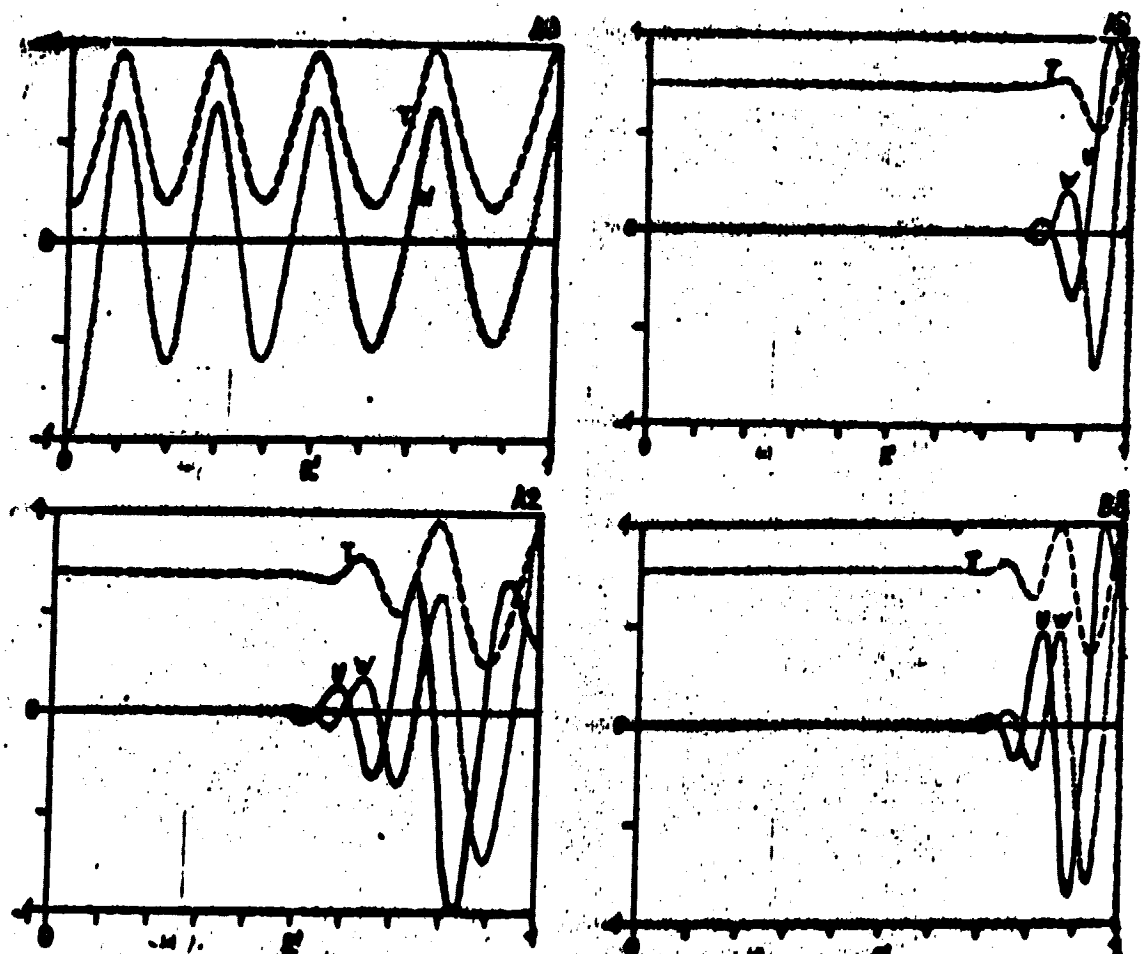

Fig. 14

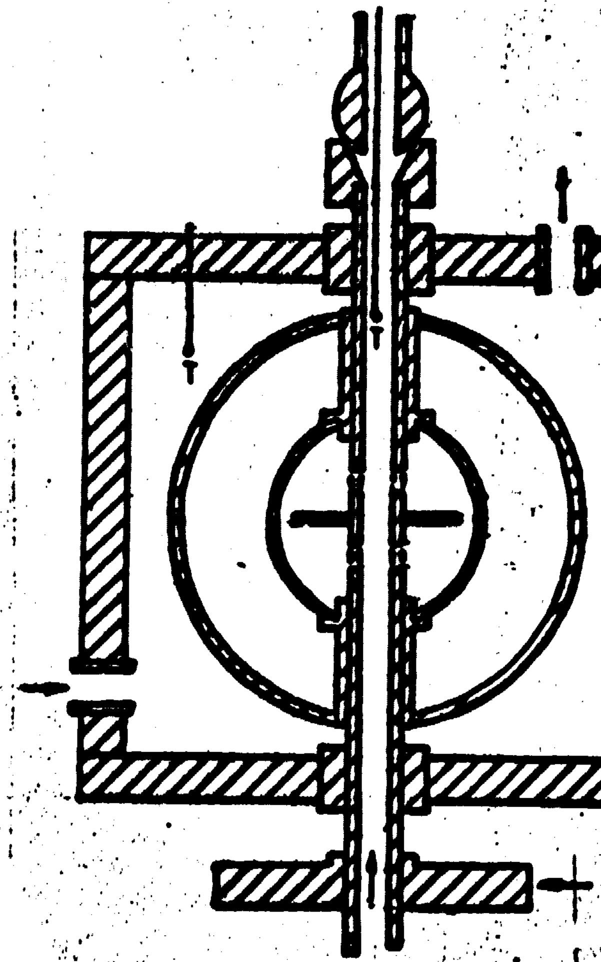

Fig 15